An Efficient Margin and Sensitivity Analysis Based Method For Calculating Voltage Collapse

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ABSTRACT

This paper concerns compute and exploiting the sensitivity of the loading margin to voltage collapse with respect to various parameters. The main idea of this paper is that after the loading margin has been computed for nominal parameters, the effect on the loading margin of altering the parameters can be predicted by Taylor series estimates.

Loading margin is a fundamental measure of proximity to voltage collapse. Linear and quadratic estimates to the variation of the loading margin with respect to any system parameter or control are derived. The accuracy of the estimates over a useful range and the ease of obtaining the linear estimate suggest that this method will be of practical value in avoiding voltage collapse.

***Keywords**: Voltage Collapse, Nominal Parameters, Bifurcation, Margin, Sensitivity*

1. INTRODUCTION

For a particular operating point, the amount of additional load in a specific pattern of load increase that would cause a voltage collapse is called the loading margin. This paper describes computing and exploiting the sensitivity of the loading margin to voltage collapse with respect to various parameters. The main idea is that after the loading margin has been computed for nominal parameters, the effect on the loading margin of altering the parameters can be predicted by Taylor series estimates. The linear Taylor series estimates are extremely quick and easy and allow many variations on the nominal case to be quickly explored. Exhaustively recomputing the point of voltage collapse instability for each parameter change is avoided.

2. NOMINAL VOLTAGE COLLAPSE MARGIN

2.1 Computing the nominal voltage collapse

The nominal point of voltage collapse is the theoretical limit of the steady state model of the power system and is not a reasonable point at which to operate the actual power system. However, by computing the nominal point of collapse, and thus the loading margin to collapse, one can assess the security of the actual system operated at a nominal stable operating point. In addition, the effects of contingencies and events on the security of the actual system can be analyzed by computing the effects of the contingencies and events on the loading margin to collapse.

The test system consists of 40 buses representing a portion of the South West Peninsula power grid and is described in [2]. For this study, transformer taps and switched compensation devices were assumed fixed. The derivations and application of the sensitivity formulas [1] require the choice of a nominal

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stable operating point at which parameters or controls are to be adjusted, and a projected pattern of load increase. Bus types are differentiated as:

- "Load bus" (PQ bus) voltage and angle vary to maintain specified real and reactive power injections.
- "Voltage controlled bus"(PV bus) reactive power output and bus angle vary to maintain specified real power injection and voltage.
- "Slack bus or reference bus" (VA bus) real and reactive power output vary to maintain specified bus voltage and angle.

3. LOADING MARGIN SENSITIVITY

This section describes and illustrates the use of loading margin sensitivities to avoid voltage collapse. The nominal stable operating point and the nominal point of collapse are described in the previous section.

The derivation of the sensitivity formulas assumes that the system equations remain fixed as parameters are varied. In particular, the limits enforced at the point of collapse are assumed to stay the same as parameters are varied. (A change in the system limits corresponds to a change in the system equations and the sensitivity based estimates using the equations valid at the nominal nose can become inaccurate when the parameters change sufficiently so that the equations change.)

For this study, when a generator represented by a 'PV' bus reaches a reactive power limit, it is converted to a 'PQ' bus, effectively changing the equilibrium equations modeling the system. In [1] the major cause for inaccuracies of the sensitivity based estimates is shown to be generator reactive power limits changing as parameters are varied.

i. Computation of linear sensitivity

The linear estimate of the change in loading margin ( ) resulting from a change to an arbitrary parameter ( ) is:

$$, \Delta p = \frac{-wFp\Delta p}{wF\lambda k} \qquad \text{....................(1)}$$

where :

- is the sensitivity of the loading margin with respect to the parameter.
- are the power system equilibrium equations (real and reactive power balance at ach bus) that apply at the nose. In particular, F accounts for the power system limits enforced at the nose.
- the derivative of F with respect to the load parameters.For constant power
- load models is a diagonal matrix with ones in the rows corresponding to buses with loads.
- , the derivative of the equilibrium equations with respect to the parameter p at the nominal nose point. The parameter can be a vector and then is a matrix.
- w, the left eigenvector corresponding to the zero eigenvalue of the system Jacobian (evaluated at a fold bifurcation is singular [11,12]).
- the unit vector in the direction of load increase. , also defines the direction in which the loading margin is measured. The direction of load increase is shown in Table 7. The denominator of (1) is a scaling factor that is the same for all parameter changes. The linear sensitivity can be improved with a quadratic estimate, derived and explained in [1].

i. Sensitivity with respect to VAR limits

Computation of the nominal voltage collapse point showed that Buses EXET0, FAWL0, and LOVE0 all encounter VAR limits. We find out from sensitivities how the loading margin to voltage collapse would change if these limits were different.

Methods

The magnitude of the components of the zero left eigenvector corresponding to reactive power injection indicates that of the three generators that encounter VAR limits between the nominal stable operating point and the point of collapse, the generator at the 132 KV bus at Exeter has the greatest influence on the margin to collapse. The loading margin corresponding to various maximum reactive power limits at the

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132KV Exeter Bus is computed by the same continuation method used to obtain the nominal fold bifurcation. The results are compared to those obtained using the linear sensitivity formula evaluated at the nominal fold bifurcation.

The nominal maximum reactive power limit at the Exeter 132KV bus is 150 MVARs. Results are obtained for a variation of 30 MVARs, or (±20%) of the nominal limit.

Results

The solid lines in Figure 4 shows the linear estimate for the loading margin variation as a function of the maximum reactive power limit at the 132 KV Exeter bus. The dots in Figure 19 represent the actual values of the loading margin as computed by the continuation method. The agreement between the linear estimates and the actual margins is excellent over the entire range.

4. CONTINGENCY RANKING FOR VOLTAGE COLLAPSE

The sensitivity formulas (1) of the previous section can be used to estimate the effects of contingencies on the margin to voltage collapse. In this case the parameter p is a vector representing the line admittance, and instead of looking at the effect of small deviations, the change in the parameter is 100%.

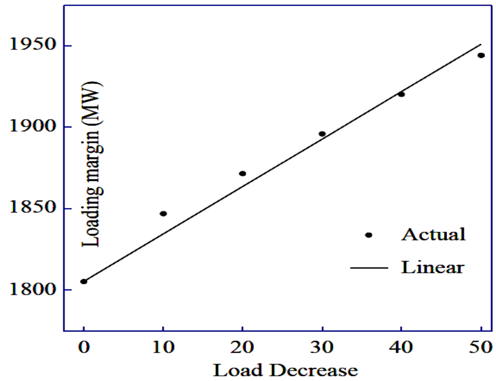

Fig.2. Effect of Load Decrease at Indian Queens on the Loading Margin to Voltage Collapse

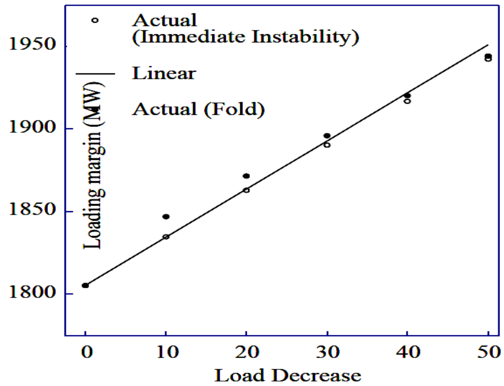

Fig.3. Effect of Load Decrease at Indian Queens on the Loading Margin to Voltage Collapse and Critical VAR Limit

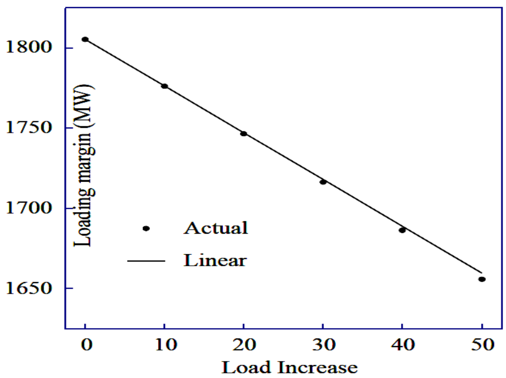

Fig.1. Effect of Load Increase at Indian Queens on the Loading Margin to Voltage Collapse

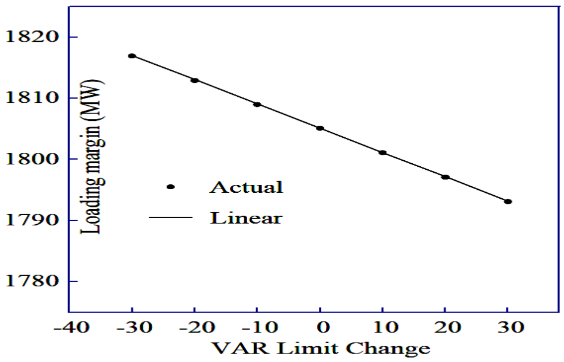

Fig.4. Effect of Variation in VAR Limits at Exeter on the Loading Margin to Voltage Collapse

Method

The estimates for the effects of contingencies were computed as described in [1,7]. The actual margins resulting from the contingencies were computed by first identifying a stable post contingency equilibrium at the base case loading and then gradually increasing the load and accounting for VAR limits until a voltage collapse due to fold bifurcation of the equilibrium equations was found.

Radial line outages are a special case in which the derived formulas do not strictly apply since the post outage network will not be connected. We suggest that the contingency list be first screened to identify radial lines, and that these outages be analyzed and ranked separately from the other contingencies.

Results

Table 1 compares the estimates to the actual margins for all non-radial line outages resulting in at least a 75 MW reduction in loading margin. The ranks correspond to the rank of each outage among all other non-radial line outages. Table 2 compares the estimates to the actual margins for all radial line outages, with ranking shown among only radial lines. The two most critical radial line outages are among the most critical line outages and are identified as so. However, the estimates for the radial line outages tend to be better than the estimates for non-radial outages, and so the moderate radial outages tend to be ranked too high when included with all line outages. Outages mis-grouped by the estimates are shown in bold face. The radial outages were all ranked correctly.

For the four outages causing less than a 10 MW change in margin, the mean error for the linear estimate was 3.0 MW and the maximum error was 4 MW. For the ten outages causing between a 10 MW and 20 MW change in margin, the mean error was 3.9 MW and the maximum error was 13 MW. For the thirteen outages causing between a 20 MW and 45 MW change in margin, the mean error was 10.4 MW and the maximum error was 26 MW. The estimates captured the thirteen worst non-radial line outages, all causing greater than 60 MW change in the margin.

As noted in [1], the major cause for inaccuracy was due to changes in the set of limits that apply at the point of collapse. This system proved to be a most challenging case, since all but nine outages forced a change in the limits applied at the nose.

As expected, often the change in VAR limits involved the HINP0 bus precariously close to a limit at the nominal point of collapse.

Table 1: Estimated and actual changes in the loading margin to fold bifurcation resulting from severe non-radial line outages (nominal loading margin = 1805 MW)

	Actual		Estimate		VAR
Line Outage	Margin MW	Change	Linear MW (rank)	Quadratic	Limited Generators
29.30.1	1464	-341 (1)	-172 (1)	-266 (1)	EXET0
4.5.1	1555	-250 (2)	-82 (4)	-133 (4)	EXET0,FAWL0
7.17.1	1635	-170 (3)	-52 (9)	-87 (7)	EXET0,FAWL0,LOVE0
8.17.1	1637	-168 (4)	-54 (8)	-89 (6)	EXET0,FAWL0,LOVE0
1.7.1	1643	-162 (5)	-129 (2)	-154 (2)	EXET0,FAWL0
1.8.1	1643	-162 (6)	-129 (3)	-154 (3)	EXET0,FAWL0
11.38.1	1650	-155 (7)	-70 (5)	-101 (5)	EXET0,FAWL0,HINP0
2.4.1	1653	-152 (8)	-48 (11)	-75 (10)	EXET0,FAWL0
3.5.1	1653	-152 (9)	-51 (10)	-80 (9)	EXET0,FAWL0
11.12.1	1700	-105 (10)	-61 (6)	-83 (8)	EXET0,FAWL0,HINP0
1.3.1	1718	-87 (11)	-56 (7)	-71 (11)	EXET0,FAWL0

5. Voltage collapse due to VAR limits

The previous sections and the theory presented in [1] associate voltage collapse of the electric power system with a fold bifurcation of the equilibrium equations used to model the system. Experience [3,4] has shown that the VAR limitations of generators are associated with voltage instability, and computational experience shows that the effect of changing(Voltage controlled bus) PV buses in the equilibrium model of the power system to (load bus) PQ buses often reduces the loading margin to voltage collapse.

In some cases, when the system loading is high, the effect of changing a PV bus to a PQ bus causes the margin to the fold bifurcation to increase. Upon application of the limit, the equilibrium point appears on the bottom half of the nose curve, and voltages increase upon increase in load [4].

The points at which changing a PV bus to a PQ bus alter the system nose curve so that the equilibrium solution is on the lower voltage branch of the new nose curve represent points at which the power system may become immediately unstable. We refer to these points as points of immediate instability, to distinguish them from fold bifurcation points. However, either a fold bifurcation point or point of immediate instability can lead to a dynamic voltage collapse.

Tables 3 & 4 show the same results as in section 4 except that the actual margins are adjusted to reflect the cases in which an immediate instability was encountered before the fold bifurcation at the nose of the curve. In all cases, the VAR limit was caused by the generator at HINP0. Those cases for which a change in margin occurs are highlighted in bold. The immediate instability caused only minor changes in ranking between outages within 5MW of each other.

Table 2: Estimated and actual changes in the loading margin to fold bifurcation resulting from radial line outages (nominal loading margin = 1805 MW)

Line Outage	Actual Margin MW	Actual Change	Estimate Linear MW (rank)	Estimate Quadratic MW (rank)	VAR Limited Generators
18.17.1	1367	-438 (1)	-265 (1)	-366 (1)	
39.38.1	1568	-237 (2)	-167 (2)	-203 (2)	EXET0,LOVE0,HINP0
6.5.1	1738	-67 (3)	-60 (3)	-62 (3)	
20.19.1	1752	-53 (4)	-46 (4)	-57 (4)	EXET0,FAWL0, LOVE0,HINP0
22.21.1	1752	-53 (5)	-46 (5)	-57 (5)	EXET0,FAWL0, LOVE0,HINP0
13.12.1	1792	-13 (6)	-19 (6)	-20 (6)	

For nearly half of the outages, instability was due to fold bifurcation, not immediate instability. All of the most serious outages were due to fold bifurcation. When the actual margin represents the distance to immediate instability and not to fold bifurcation, the margin to fold bifurcation is noted in parentheses. In all cases, fold bifurcation occurs within 11 MW of immediate instability.

Table 3: Estimated and actual changes in the loading margin to voltage collapse resulting from severe non-radial line outages (nominal loading margin = 1805 MW)

Line Outage	Actual Margin MW	Actual Change	Estimate VAR (N) MW (rank)	Estimate Fold (W) MW (rank)	VAR Limited Generators
29.30.1	1464	-341 (1)	-172 (1)	-148 (1)	EXET0
4.5.1	1555	-250 (2)	-82 (4)	-80 (4)	EXET0,FAWL0
7.17.1	1635	-170 (3)	-52 (9)	-49 (10)	EXET0,FAWL0,LOVE0
8.17.1	1637	-168 (4)	-54 (8)	-51 (8)	EXET0,FAWL0,LOVE0
1.7.1	1643	-162 (5)	-129 (2)	-127 (2)	EXET0,FAWL0
1.8.1	1643	-162 (6)	-129 (3)	-126 (3)	EXET0,FAWL0
11.38.1	1650	-155 (7)	-70 (5)	-77 (5)	EXET0,FAWL0,HINP0
2.4.1	1653	-152 (8)	-48 (11)	-47 (11)	EXET0,FAWL0
3.5.1	1653	-152 (9)	-51 (10)	-51 (9)	EXET0,FAWL0
11.12.1	**1699 (1700)**	-106 (10)	-61 (6)	-69 (6)	EXET0,FAWL0,HINP0
1.3.1	1718	-87 (11)	-56 (7)	-56 (7)	EXET0,FAWL0

6. CONTINGENCY RANKING FOR VOLTAGE COLLAPSE DUE TO VAR LIMITS

The sensitivity computations can be easily extended to the case were the voltage collapse is an immediate instability due to a VAR limit rather than a fold bifurcation. The derivations of the sensitivity formulas in [1] required the description of a manifold in which each point on the manifold corresponded to a point of fold bifurcation. The normal vector to this surface is defined by the zero left eigenvector of the system Jacobian. Similarly we can construct a manifold in which each point corresponds to the point at which a particular generator is at a VAR limit. The normal vector to this surface can then be used in the sensitivity formulas to compute the sensitivity of the margin to encountering a VAR limit.

Table 4: Estimated and actual changes in the loading margin to voltage collapse resulting from radial line outages (nominal loading margin = 1805 MW)

Line Outage	Actual Margin MW	Actual Change	Estimate Linear MW (rank)	Estimate Quadratic MW (rank)	VAR Limited Generators
18.17.1	1367	-438 (1)	-265 (1)	-366 (1)	
39.38.1	1568	-237 (2)	-167 (2)	-203 (2)	EXET0,LOVE0,HINP0
6.5.1	1738	-67 (3)	-60 (3)	-62 (3)	
20.19.1	**1750 (1752)**	-55 (4)	-46 (4)	-57 (4)	EXET0,FAWL0, LOVE0,HINP0
22.21.1	**1750 (1752)**	-55 (5)	-46 (5)	-57 (5)	EXET0,FAWL0, LOVE0,HINP0
13.12.1	1792	-13 (6)	-19 (6)	-20 (6)	

7. COMPUTATION OF SENSITIVITY

When the voltage collapse is identified with the fold bifurcation of the equilibrium model, the left zero eigenvector can be used to compute the normal vector to the surface of bifurcation points in parameter space. Similarly, when the voltage collapse is identified with the immediate instability due to application of a VAR limit, there is a normal vector in parameter space to the surface of points at which the critical Q limit is reached. In short, when equation (1) is evaluated with the vector w computed at the critical VAR limit point (as opposed to at the fold bifurcation point), reflects the linear estimate of the change in margin to the VAR limit for the change in parameter . Table 17 compares the VAR limit normal vector N to the zero left eigenvector W. The angle between N and W is 4.6 degrees. Note that it is considerably easier to compute N than W. There is no need to compute a Hessian term and computation of N does not require a good initial guess.

When the sensitivity computations and contingency rankings were repeated using N in place of W, no significant changes were observed. Tables 18 and 19 compare the linear estimates for the change in margin resulting from the lines outages computed with both the left zero eigenvector, W, at the original fold bifurcation point, and the normal vector to the VAR limit set, N, with the actual margins to instability for the original system. Note that there is very little difference in the estimates, and the top twelve contingencies are ranked the same except for two contingencies with a difference of less than 1 MW are switched. In each case limited by fold bifurcation, the estimate computed with the zero left eigenvector is better than the estimate computed with the normal vector. For the one contingency of the top twelve limited by immediate instability, the VAR limit normal vector is superior. Table 19 shows the less critical contingencies which tend to be limited by immediate instability as opposed to fold bifurcation. The estimates are very similar, seldom differing by more than 5 MW. However, the estimates computed with the normal vector to the VAR limit set are better than those computed with the zero left eigenvector for the cases that were limited by immediate instability.

7. Conclusions

Table 5: Comparison of linear estimated changes in the loading margin to voltage collapse resulting from severe non-radial line outages (nominal loading margin = 1805 MW)

	Actual		Estimate		VAR
Line Outage	Margin MW	Change	Linear MW (rank)	Quadratic	Limited Generators
29.30.1	1464	-341 (1)	-172 (1)	-266 (1)	EXET0
4.5.1	1555	-250 (2)	-82 (4)	-133 (4)	EXET0,FAWL0
7.17.1	1635	-170 (3)	-52 (9)	-87 (7)	EXET0,FAWL0,LOVE0
8.17.1	1637	-168 (4)	-54 (8)	-89 (6)	EXET0,FAWL0,LOVE0
1.7.1	1643	-162 (5)	-129 (2)	-154 (2)	EXET0,FAWL0
1.8.1	1643	-162 (6)	-129 (3)	-154 (3)	EXET0,FAWL0
11.38.1	1650	-155 (7)	-70 (5)	-101 (5)	EXET0,FAWL0,HINP0
2.4.1	1653	-152 (8)	-48 (11)	-75 (10)	EXET0,FAWL0
3.5.1	1653	-152 (9)	-51 (10)	-80 (9)	EXET0,FAWL0
11.12.1	1699 (1700)	-106 (10)	-61 (6)	-83 (8)	EXET0,FAWL0,HINP0
1.3.1	1718	-87 (11)	-56 (7)	-71 (11)	EXET0,FAWL0

The nominal voltage collapse occurred at an equilibrium at which a critical VAR output was precariously close to a limit. It was expected that line outages as well as small changes in parameters would cause the system to encounter the limit prior to voltage collapse. This phenomena had previously been associated with inaccuracies in the estimates and rankings.

For this case, it was shown that the VAR limited changes produced a noticeable but negligible effect on the estimates. In addition, the formulas were shown to perform well estimating the effect on the loading margin of altering a generator maximum VAR limit, illustrating a new application for the methods. This chapter also tested a new implementation of the sensitivity formulas derived in Chapter 4 using the node voltage form of the equilibrium equations. Contingency analysis was performed by evaluating the sensitivity formulas for changes in the impedance matrix resulting from line outages. Finally, since voltage collapse can be precipitated by generator power limits, a new estimate was tested using the normal vector to the critical VAR limit in place of the zero left eigenvector at a fold bifurcation. The results were comparable. However, the VAR normal vector is easier to compute than the left zero eigenvector and also does not require that a continuation method locate an exact fold bifurcation point.

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