Double Diffusive Convection in a Couple Stress Fluid Saturated Rotating Anisotropic Porous Layer with Internal Heating and Soret Effect

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Abstract

In this paper, the effect of internal heat source and Soret effect has been investigated on double diffusive convection in a rotating anisotropic porous medium saturated with a couple stress fluids, heated and salted from below. Linear stability analysis has been performed by using Normal mode technique and for nonlinear analysis, minimal representation of Fourier series up to two terms has been considered. The modified Darcy model, which includes the time derivative term and Coriolis term, has been employed in the momentum equation. The effect of Taylor number, couple stress parameter, solute Rayleigh number, internal heat source parameter, Lewis number, Darcy-Prandtl number, thermal and mechanical anisotropy parameter on the stationary and oscillatory modes of convection has been obtained and shown graphically, Also the heat and mass transports are obtained in terms of the Nusselt number and Sherwood number respectively, and shown graphically.

1. INTRODUCTION

A well-known phenomenon that involves coupled heat and mass transfer is the thermal energy flux that is generated by concentration gradients is called the Dufour (diffusion-thermal) effect. The Soret (thermo-diffusion) effect is the effect of the temperature gradient on mass flux. Swiss scientist J. Soret was the first to study the thermo diffusion in 1879. Hurle and Jakeman [1] performed theoretical study of Soret driven thermosolutal convection in a binary fluid mixture. Gaikwad et. al. [2] studied linear and non-linear double diffusive convection in a fluid saturated anisotropic porous layer with cross-diffusion effects. Malashetty et.al. [3] studied linear and nonlinear double diffusive convection in a fluid saturated porous layer with cross diffusion effect. Recently, Altawallbeh et al. [4] studied double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect and internal heat source.

Internal heat generation arises in many important situations, including reactor safety analyses, metal waste that is produced by spent nuclear fuel, fire and combustion studies, and the storage of radioactive materials. The study concerning internal heat source in porous media was provided by Tveitereid [5], who studied thermal convection in a horizontal porous layer with internal heat sources. Hill [6] performed linear and nonlinear analyses on the double-diffusive convection in a porous layer with a concentration based internal heat source. Bhadauria et al. [7]-[9] studied the effect of internal heating on double diffusive convection in a couple stress fluid saturated anisotropic porous medium.

The study of double diffusive convection in a rotating porous media is importance both theoretically and due to its practical applications in engineering. Some of the important areas of application in engineering include the food and chemical process, solidification and centrifugal casting of metals, rotating machinery, petroleum industry and biomechanics problems. Chakrabarti and Gupta [10] have analyzed the nonlinear thermohaline convection in a rotating porous medium. The effect of rotation on linear and nonlinear double diffusive convection in a sparsely packed porous medium was studied by Rudraiah et.al.[11]. Malashetty et al. performed double diffusive convection in a Darcy porous medium saturated with couple stress and also carried out effect of rotation on the onset of double diffusive convection in a Darcy porous medium saturated with couple stress fluid [12]-[13]. M.S. Malashetty and Rajasekhar Heera [14] studied the effect of rotation on the onset of double diffusive convection in a horizontal anisotropic porous layer. Gaikwad [15] have studied linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in presence of Soret effect. Sulochana et.al [16] performed the onset of double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer. Bhadauria et al. [17] studied cross diffusion convection in a Newtonian fluid saturated rotating porous medium. Very first study on double diffusive convection in porous media mainly concerns with linear stability analysis, and was performed by Nield [18].

With the growing importance of non-Newtonian fluids with suspended particles in modern technology and industries, the investigation of such fluids is desirable. Applications of such fluids are found in extrusion of polymer fluids in industry, exotic suspension, fluid film lubrication, solidification of liquid crystals, cooling of metallic plate in bath, colloidal and suspension solutions. In the category of non-Newtonian, couple stress fluids have specific features, such as polar effect. The theory of polar fluids and related theories are models for fluids whose microstructure is mechanically significant. Theory for couple stress fluid was proposed by Stokes [19], which is a simpler polar fluid theory and shows all the important features and effect of such fluids that occur inside a deforming continuum. Stabilizing effect of couple stress parameter is reported in the works of Sharma and Thakur [20], who investigated thermal instability in an electrically conducting couple stress fluid with magnetic field. Sunil et al. [21] studied the effect of suspended particles on double diffusive convection in a couple stress fluid saturated porous medium, Sharma and Sharma [22] investigated the effect of suspended particles on couple stress fluid, heated from below, in the presence of rotation and magnetic field. Malashetty et al. [23] did an analytical study of linear and nonlinear double diffusive convection with Soret effect in couple stress liquids. Gaikwad et al. [24] performed linear stability analysis of double diffusive convection in a horizontal sparsely packed rotating anisotropic porous layer in the presence of Soret effect. Malashetty and Kollur [25] investigated the onset of double diffusive convection in a couple stress fluids saturated anisotropic porous layer. Shivkumara [26] carried out linear and non linear stability analysis of double diffusive convection in a couple stress fluid saturated porous layers.

In the present literature, no work is available related to the rotation with an internal heat source and Soret parameter. Therefore, in the present study stability analysis of Soret effect and internal heating effect on double diffusive convection in a rotating anisotropic porous medium saturated with a couple stress fluids has been investigated.

2. GOVERNING EQUATION

We consider a couple stress fluid saturated porous medium, confined between two infinitely extended horizontal planes, at $z=0$ and $z=d$, heated from below and cooled from above. Darcy model has been employed in the momentum equation. Further, an internal heat source term has been included in the energy equation. A Cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis as vertically upward. An adverse temperature gradient is applied across the porous layer and the lower and upper planes are kept at temperature $T_0+\Delta T$, and T_0, and concentration $S_0+\Delta S$, and S_0 respectively. The governing equations are given by

$$\nabla . q = 0 \tag{1}$$

$$\frac{1}{\phi}\frac{\partial q}{\partial t} = -\nabla p + \rho_0 g\left(\beta_T T - \beta_S S\right) - \frac{2\rho_0}{\phi}(\Omega \times q) - \left(\mu - \mu_c \nabla^2\right) q_a \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (q\cdot\nabla)T = \nabla(\kappa_T \cdot \nabla T) + QT - w\frac{\partial T_b}{\partial z} \tag{3}$$

$$\phi\frac{\partial S}{\partial t} + (q\cdot\nabla)S = w\frac{\Delta S}{d} + \kappa_S \nabla^2 S + K_{21}\nabla^2 T \tag{4}$$

$$\rho = -\rho_0\left[\beta_T T - \beta_S S\right] \tag{5}$$

The physical variables have their usual meaning. The externally imposed thermal and Solutal boundary conditions are given by

$$T = T_0 + \Delta T;\ \ at\ \ z = 0\ \ and\ \ T = T_0;\ \ at\ z = d;$$
$$S = S_0 + \Delta S;\ \ at\ \ z = 0\ \ and\ \ S = S_0;\ \ at\ z = d; \tag{7}$$

2.1. Basic Solution

At this state, the velocity, pressure, temperature and density profiles are given by

$$q_b = 0, p = p_b(z), T = T_b(z), S = S_b(z), \rho = \rho_b(z). \tag{8}$$

Substituting Eq. (8) in Eq. (1-4), we get the following equations:

$$\frac{dp_b}{dz} = -\rho_b g, \tag{9}$$

$$\kappa_T\frac{d^2T_b}{dz^2} + QT = 0, \tag{10}$$

$$\frac{d^2S_b}{dz^2} = 0, \tag{11}$$

the solution of Eq. (10) and Eq. (11) subject to the boundary conditions (7), are given by

$$T_b = T_0 + \Delta T\,\frac{\sin\sqrt{R_i}\left(1-\frac{z}{d}\right)}{\sin\sqrt{R_i}}. \tag{12}$$

$$S_b = S_0 + \Delta S\left(1-\frac{z}{d}\right) \tag{13}$$

Now, we superimpose finite amplitude perturbations on the basic state in the form:

$$q = q_b + q',\ T = T_b + T',\ p = p_b + p',\ S = S_b + S',\ \rho = \rho_b + \rho', \tag{14}$$

The resulting equations are then non-dimensionalized using the following transformations;

$$(x', y', z') = (x^*, y^*, z^*)d,\ t' = t^*\left(\frac{\gamma d^2}{\kappa_{Tz}}\right),\ q' = \frac{\kappa_{Tz}}{d}q^*,$$

$$(u,v,w) = (u^*, v^*, w^*)\left(\frac{\kappa_{Tz}}{d}\right),\ T' = (\Delta T)T^*,\ S' = (\Delta S)S^* \tag{15}$$

the pressure term is eliminated by taking curl twice of the momentum equation. The non-dimensionalized equations (on dropping the asterisks for simplicity) are obtained as,

$$\left[\left(\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2 + \left(\nabla_h^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)(1 - C\nabla^2)\right)\left(\frac{1}{Pr_D}\frac{\partial}{\partial t} + \frac{1}{\xi} - C\nabla^2\right) + T_a\frac{\partial^2}{\partial z^2}\right]w$$
$$-\left(\frac{1}{Pr_D}\frac{\partial}{\partial t} + \frac{1}{\xi} - c\nabla^2\right)\left(Ra_T\nabla_h^2 T - Ra_S\nabla_h^2 S\right) = 0 \tag{16}$$

$$\left(\frac{\partial}{\partial t} - \left(\eta\nabla_h^2 + \frac{\partial^2}{\partial z^2}\right) - R_i\right)T + w\frac{\partial T_b}{\partial z} = 0 \tag{17}$$

$$\left[\frac{\partial}{\partial t} - \nabla^2\frac{1}{L_e}\right]S - S_r\nabla^2 T - w = 0 \tag{18}$$

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where $Pr_D = \dfrac{\phi\gamma\nu d^2}{\kappa_T k}$ is Darcy-Prandtl number, $Ra_T = \dfrac{\beta_T g \Delta T K_z d}{\nu\kappa_{Tz}}$ is the thermal Rayleigh number, $Ra_S = \dfrac{\beta_S g \Delta S K_z d}{\nu\kappa_{Tz}}$ is the solute Rayleigh number, $R_i = \dfrac{Qd^2}{\kappa_{Tz}}$ is the internal Rayleigh parameter, $T_a = \left(\dfrac{2\Omega K_z}{\mu\phi}\right)^2$ is Taylor number, $\eta = \dfrac{\kappa_{Tx}}{\kappa_{TZ}}$ is thermal anisotropy parameter, $L_e = \dfrac{\kappa_{TZ}}{\kappa_S}$ is Lewis number, $S_r = \dfrac{K_{21}\Delta T}{\kappa_{TZ}\Delta S}$ the Soret parameter, $\xi = \dfrac{K_x}{K_z}$ mechanical anisotropy parameter. The above system will be solved by considering stress free and isothermal boundary conditions as given below:

$$w = \frac{\partial^2 w}{\partial z^2} = T = S = 0 \quad on \quad z = 0, z = 1. \qquad (19)$$

3. LINEAR STABILITY ANALYSIS

In order to do linear stability analysis, we solve the Eigen value problem defined by Eq.(16)-(18) subject to the boundary condition by (19), using time dependent periodic disturbance in horizontal plane as

$$(w, T, S) = (W, \Theta, \phi)\, exp\left(i\left(lx + my\right) + \sigma t\right), \qquad (20)$$

Where *l*, *m* are horizontal wave numbers and $\sigma = \sigma_r + i\sigma_i$, growth rate. Substituting Eq. (20) into the linerized Eq. (16)-(18), we obtain

$$\left[\left(\frac{\sigma}{Pr_D}\left(D^2-a^2\right)+\left(\frac{D^2}{\xi}-a^2\right)\left(1-C\left(D^2-a^2\right)\right)\right)\left(\frac{\sigma}{Pr_D}+\frac{1}{\xi}-C\left(D^2-a^2\right)\right)+T_a D^2\right]W$$
$$=\left(\frac{\sigma}{Pr_D}+\frac{1}{\xi}-C\left(D^2-a^2\right)\right)\left(-a^2 Ra_T\,\Theta+a^2 Ra_S\,\phi\right) \qquad (21)$$

$$[\,\sigma-(D^2 - \eta a^2) - Ri]\,\Theta - W = 0 \qquad (22)$$

$$\left[\sigma + \frac{D^2 - a^2}{L_e}\right]\phi - W - (D^2 - a^2) S_r \Theta = 0. \qquad (23)$$

Where D=d/dz and $a^2 = l^2 + m^2$, the boundary conditions (19) now becomes

$$W = \frac{\partial^2 W}{\partial z^2} = \Theta = \phi = 0 \quad on \quad z = 0, z = 1. \qquad (24)$$

We assume the solution W, Θ, ϕ

$$\left(W, \Theta, \phi\right) = \left(W_0, \Theta_0, \phi_0\right) \mathrm{Sin}\, n\pi z \qquad (n=1,2,3........),$$

The most unstable mode corresponds to n = 1 (fundamental mode). Therefore, substituting above equation with n = 1 into Eq.(21)-(23), we obtain a matrix form $A\,X = 0$

$$\begin{pmatrix} \left(\frac{\sigma\delta^2}{Pr_D}+\delta_1^2\left(C\delta^2+1\right)\right)\left(\frac{\sigma}{Pr_D}+\frac{1}{\xi}+C\delta^2\right)+T_a\pi^2 & \left(\frac{\sigma}{Pr_D}+\frac{1}{\xi}+C\delta^2\right)a^2 Ra_T & -\left(\frac{\sigma}{Pr_D}+\frac{1}{\xi}+C\delta^2\right)a^2 Ra_S \\ -1 & \left(\sigma+\delta^2-R_i\right) & 0 \\ -1 & S_r\delta^2 & \sigma+\frac{\delta^2}{Le} \end{pmatrix}\begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \end{pmatrix} = 0. \qquad (25)$$

The thermal Rayleigh number can be obtained as

$$Ra_T = \frac{(\sigma+\delta_2^2-R_i)}{a^2}\left[\left(\frac{\sigma\delta^2}{Pr_D}+\delta_1^2(C\delta^2+1)\right)+\frac{T_a\pi^2}{\frac{\sigma}{Pr_D}+\frac{1}{\xi}+C\delta^2}\right]$$
$$+\frac{(\sigma+\delta_2^2-R_i)-\delta^2 S_r}{\left(\sigma+\frac{1}{L_e}\right)\left(\frac{\sigma}{Pr_D}+\frac{1}{\xi}+C\delta^2\right)}Ra_S \qquad (26)$$

where $\delta^2 = \pi^2 + a^2$ $\delta_1^2 = \dfrac{\pi^2}{\xi} + a^2$, $\delta_2^2 = \pi^2 + \eta a^2$

The growth rate σ is in general a complex quantity such that $\sigma = \sigma_r + i\,\sigma_i$. The system with $\sigma_r < 0$ is always stable, while for $\sigma_r > 0$ it will become unstable. For neutral stability state $\sigma_r = 0$

3.1 Stationary State

We now set $\sigma = 0$ at the margin of stability. The expression of the thermal Rayleigh number for stationary mode of convection is as given below:

$$Ra_T^{st} = \frac{(\delta^2 - R_i)}{a^2}\left[\delta_1^{\,2}(1+C\delta^2) + \frac{T_a\pi^2}{\frac{1}{\xi}+C\delta^2}\right] + \frac{\left((\delta_2^{\,2} - R_i) - \delta^2 S_r\right)L_e}{\delta^2\left(\frac{1}{\xi}+C\delta^2\right)}Ra_S \tag{27}$$

It is important to note that the critical wave number $a = a_c^{St}$ depends on the couple stress parameter and Taylor number. In the absence of Taylor number i.e. $T_a = 0$ Eq. (27) gives

$$Ra_T^{st} = \frac{(\delta^2 - R_i)}{a^2}\left[\delta_1^{\,2}(1+C\delta^2) + \frac{\left((\delta_2^{\,2} - R_i) - \delta^2 S_r\right)L_e}{\delta^2\left(\frac{1}{\xi}+C\delta^2\right)}Ra_S\right] \tag{28}$$

For isotropic porous media when $\xi = \eta = 1$ and the system without internal-heating, i.e., $R_i = 0$ we get

$$Ra_T^{st} = \frac{(\pi^2+a^2)^2}{a^2}(1+C\delta^2) + L_e Ra_S \frac{\left((\pi^2+a^2) - (\pi^2+a^2)S_r\right)}{\delta^2\left(1+C\delta^2\right)}, \tag{29}$$

In case of no solute Rayleigh number i.e. $Ra_S = 0$

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2}(1+C\delta^2) \tag{30}$$

which is the same as obtained by Shivkumara et al.[26]. When C=0 (i.e. Newtonian fluid)

$$Ra_T^{st} = \frac{(\pi^2 + a^2)^2}{a^2} \tag{31}$$

which has the critical value $Ra_T^{st} = 4\pi^2$ for $a_c^{st} = \pi$ and which are the classical results obtained by Horton and Roger [27] and Lapwood [28] for single component fluid in porous layer.

3.2 Oscillatory State

For the oscillatory mode of convection, we set $\sigma = i\sigma_i$ in Eq. (27) and clear the complex quantities from the denominator, to obtain

$$Ra_T^{osc} = \Delta_1 + i\sigma_i\Delta_2 \tag{32}$$

Where

$$\Delta_1 = \frac{d_1 d_3 + \sigma^2 d_2 d_4}{d_3^{\,2} + \sigma^2 d_4^{\,2}}, \quad \Delta_2 = \frac{d_2 d_3 - d_1 d_4}{d_3^{\,2} + \sigma^2 d_4^{\,2}}$$

For oscillatory onset $\Delta_2 = 0$ and $\sigma_i \neq 0$, where σ is the oscillatory frequency which is not given for brevity.

Where

$$A_1 = T_a\pi^2 \Pr_D\left(\delta_2^{\,2} - R_i\right) - \sigma^2\delta^2\left(\frac{\left(\delta_2^{\,2} - R_i\right)}{P_{rD}} + \frac{1}{\xi} + C\delta^2\right) + \delta_1^{\,2}\Pr_D\left(1+C\delta^2\right)\left(\left(\delta_2^{\,2} - R_i\right)\left(\frac{1}{\xi}+C\delta^2\right) - \frac{\sigma^2}{\Pr_D}\right) \tag{33}$$

$$A_2 = \delta^2\left(\left(\delta_2^{\,2} - R_i\right)\left(\frac{1}{\xi}+C\delta^2\right) - \frac{\sigma^2}{\Pr_D}\right) + T_a\pi^2\Pr_D + \delta_1^{\,2}\Pr_D\left(1+C\delta^2\right)\left(\frac{\left(\delta_2^{\,2} - R_i\right)}{\Pr_D} + \frac{1}{\xi} + C\delta^2\right) \tag{34}$$

$$B_1 = Ra_S\left(\delta_2^{\,2} - R_i - \delta^2 S_r\right) \tag{35}$$

$$B_2 = Ra_S \tag{36}$$

$$C_1 = a^2 Pr_D\left(\frac{1}{\xi} + C\delta^2\right) \tag{37}$$

$$C_2 = a^2 \tag{38}$$

$$C_3 = \frac{\delta^2}{L_e}\left(\frac{1}{\xi} + C\delta^2\right) - \frac{\sigma^2}{Pr_D} \tag{39}$$

$$C_4 = \frac{\delta^2}{L_e Pr_D} + \frac{1}{\xi} + C\delta^2 \tag{40}$$

$$d_1 = A_1C_3 + B_1C_1 - \sigma^2\left(A_2C_4 + B_2C_2\right) \tag{41}$$

$$d_2 = A_2C_3 + B_2C_1 + A_1C_4 + B_1C_2 \tag{42}$$

$$d_3 = C_1C_3 - \sigma^2 C_2C_4 \tag{43}$$

$$d_4 = C_2C_3 + C_1C_4 \tag{44}$$

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We have the necessary expression for oscillatory Rayleigh number as:

$$Ra_T^{osc} = \Delta_1 \tag{45}$$

4. NONLINEAR STABILITY ANALYSIS

In this section, nonlinear stability has been studied using minimal truncated Fourier series. For simplicity, we consider only two dimensional rolls, so that all the physical quantities are independent of y. We introduce the stream function ψ as $u = \frac{\partial \psi}{\partial z}$, w= -$\frac{\partial \psi}{\partial x}$, then taking curl to eliminate pressure term from Eq.(2), to get

$$\left(\frac{1}{Pr_D}\frac{\partial}{\partial t}\nabla^2 + \left(\frac{\partial 2}{\partial x2} + \frac{1}{\xi}\frac{\partial 2}{\partial z2}\right)\left(1 - C\nabla^2\right)\right)\psi - \left(T_a\right)^{\frac{1}{2}}\frac{\partial V}{\partial z} + Ra_T\frac{\partial T}{\partial x} - Ra_S\frac{\partial S}{\partial x} = 0 \tag{46}$$

$$\left(\frac{1}{Pr_D}\frac{\partial}{\partial t} + \left(\frac{1}{\xi} - C\nabla^2\right)\right)V + \left(T_a\right)^{\frac{1}{2}}\frac{\partial \psi}{\partial z} = 0 \tag{47}$$

$$\left(\frac{\partial}{\partial t} - \left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - R_i\right)\right)T - \frac{\partial(\psi, T)}{\partial(x,z)} + \frac{\partial \psi}{\partial x} = 0 \tag{48}$$

$$\left(\frac{\partial}{\partial t} - L_e^{-1}\nabla^2\right)S - \frac{\partial(\psi, S)}{\partial(x,z)} + \frac{\partial \psi}{\partial x} - S_r\nabla^2 T = 0 \tag{49}$$

It is to be noted that the effect of nonlinearity is to distort the temperature concentration fields through the interaction of ψ, T and S. As a result a component of the form $Sin(2\pi z)$ will be generated. A minimal Fourier series which describes the finite amplitude convection is given by

$$\psi = M_0(t) sin(ax) sin(\pi z), \tag{50}$$

$$T = M_1(t) cos(ax) sin(\pi z) + M_2(t) sin(2\pi z), \tag{51}$$

$$S = M_3(t) cos(ax) sin(\pi z) + M_4(t) sin(2\pi z) \tag{52}$$

$$V = F(t)\sin(ax)\cos(\pi z) + G(t) sin(2\pi z) \tag{53}$$

Where the amplitudes $M_0(t)$, $M_1(t)$, $M_2(t)$, $M_3(t)$, $M_4(t)$, $F(t)$, $G(t)$ are functions of time and are to be determined. Substituting above expressions in Eq.(46)-(49) and equating the like terms, the following set of nonlinear autonomous differential equations is obtained

$$\frac{dX}{dt} = D \tag{54}$$

where $X = \left(M_0, M_1, M_2, M_3, M_4, F, G\right)^T$

$$D = \left(D_0, D_1, D_2, D_3, D_4, D_5, D_6\right)^T$$

$$D_0 = -\frac{Pr_D}{\delta^2}\left[\delta_1^2(1 + C\delta^2)M_0 - \pi\left(T_a\right)^{\frac{1}{2}}F + aRa_T M_1 - aRa_S M_3\right] \tag{55}$$

$$D_1 = -\left[aM_0 + \pi a M_0 M_2 + \left(\delta_2^{\,2} - R_i\right)M_1\right] \tag{56}$$

$$D_2 = -\left[\left(4\pi^2 - R_i\right)M_2 - \frac{\pi a}{2}M_0 M_1\right] \tag{57}$$

$$D_3 = -\left[aM_0 + \pi a M_0 M_2 + \delta^2\frac{1}{L_e}M_3 + S_r\delta^2 M_1\right] \tag{58}$$

$$D_4 = -\left[4\pi^2\frac{1}{L_e}M_4 - \frac{\pi a}{2}M_0 M_3 + 4\pi^2 S_r M_2\right] \tag{59}$$

$$D_5 = -\Pr_D\left[\left(\frac{1}{\xi} + C\delta^2\right)F + \pi\left(T_a\right)^{\frac{1}{2}}M_0\right] \tag{60}$$

$$D_6 = -\Pr_D\left(\frac{1}{\xi} + C\delta^2\right)G \tag{61}$$

4.1. *Steady Finite Amplitude Motions*

For steady state finite amplitude convection we have to set left hand side of the Eq. (54) to zero.

$$\delta_1^2(1 + C\delta^2)M_0 - \pi\left(T_a\right)^{\frac{1}{2}}F + aRa_T M_1 - aRa_S M_3 = 0 \tag{62}$$

$$aM_0 + \pi a M_0 M_2 + \left(\delta_2^{\,2} - R_i\right)M_1 = 0 \tag{63}$$

$$\left(4\pi^2 - R_i\right)M_2 - \frac{\pi a}{2}M_0 M_1 = 0 \tag{64}$$

$$aM_0 + \pi a M_0 M_2 + \delta^2\frac{1}{L_e}M_3 + S_r\delta^2 M_1 = 0 \tag{65}$$

$$4\pi^2\frac{1}{L_e}M_4 - \frac{\pi a}{2}M_0 M_3 + 4\pi^2 S_r M_2 = 0 \tag{66}$$

$$\left(\frac{1}{\xi} + C\delta^2\right)F + \pi\left(T_a\right)^{\frac{1}{2}}M_0 = 0 \tag{67}$$

$$\Pr_D\left(\frac{1}{\xi}+C\delta^2\right)G=0 \tag{68}$$

Numerical method was used to solve the above nonlinear differential equation to find the amplitudes on solving for the amplitudes in terms of M_0, we obtain M_1, M_2, M_3, M_4 as

$$M_1=-\frac{2a\left(4\pi^2-R_i\right)M_0}{a^2M_0{}^2\pi^2+2\left(4\pi^2-R_i\right)\left(\delta_2{}^2-R_i\right)} \tag{69a}$$

$$M_2=-\frac{a^2\pi M_0{}^2}{a^2M_0{}^2\pi^2+2\left(4\pi^2-R_i\right)\left(\delta_2{}^2-R_i\right)} \tag{69b}$$

$$M_3=-\frac{\left(8aM_0L_e\right)\left(a^2\pi^2M_0^2\left(1+L_eS_r\right)+2\left(4\pi^2-R_i\right)\left(\delta_2^2-R_i-S_r\delta^2\right)\right)}{\left(a^2M_0^2L_e^2+8\delta^2\right)\left|\left(a^2M_0^2\pi^2+2\left(4\pi^2-R_i\right)\left(\delta_2^2-R_i\right)\right)\right|} \tag{69c}$$

$$M_4=-\frac{\left(a^2M_0^2L_e\right)\left(\pi^2\left(a^2M_0^2L_e-8\delta^2S_r\right)+2L_e\left(4\pi^2-R_i\right)\left(\delta_2^2-R_i-S_r\delta^2\right)\right)}{\pi\left(a^2M_0^2L_e^2+8\delta^2\right)\left|\left(a^2M_0^2\pi^2+2\left(4\pi^2-R_i\right)\left(\delta_2^2-R_i\right)\right)\right|} \tag{69d}$$

In the study of this type problem, quantification of heat and mass transport is very important.

If H and J are the rate of heat and mass transport per unit area. Then

$$H=-K_T\langle\frac{\partial T_{total}}{\partial z}\rangle_{z=0} \tag{70a}$$

$$J=-K_S\langle\frac{\partial T_{total}}{\partial z}\rangle_{z=0}-K_{21}\langle\frac{\partial S_{total}}{\partial z}\rangle_{z=0} \tag{70b}$$

where the angular bracket corresponds to a horizontal average and

$$T_{total}=T_0-\Delta T\,\frac{z}{d}+T(x,z,t) \tag{70c}$$

$$S_{total}=S_0-\Delta S\,\frac{z}{d}+S(x,z,t) \tag{70d}$$

Substituting Eq.(50)-(51) into Eqs.(69c,d) and using the resultant Eqs.(69a,b), we get

$$H=\frac{K_T\Delta T}{d}(1-2\pi M_2) \tag{71a}$$

$$J=\frac{K_S\Delta S}{d}[(1-2\pi M_4)+S_rL_e(1-2\pi M_2)] \tag{71b}$$

Substituting the value of Eq. (71a,b), The Nusselt number and Sherwood number are obtained as

$$Nu=\frac{H}{\frac{K_T\Delta T}{d}}=(1-2\pi M_2) \tag{72}$$

$$Sh=\frac{J}{\frac{K_S\Delta S}{d}}=(1-2\pi M_4)+S_rL_e(1-2\pi M_2)$$

Substituting M_2, M_4 in (72), the expressions for Nu and Sh are obtained as above.

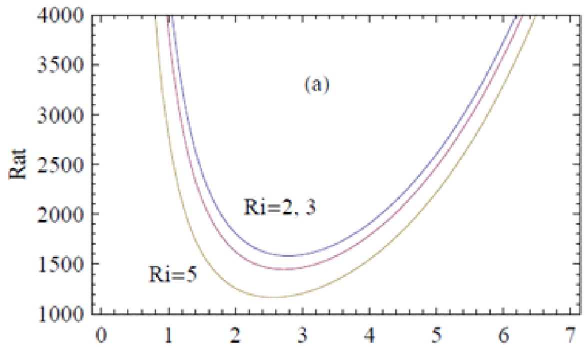

Fig. 1: Variation of Stationary Rayleigh number with wave number for the different values of R_i

5. RESULTS AND DISCUSSION

Effects of Soret parameter and internal heat source have been studied on the double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer using linear and nonlinear stability analyses. In this section, we discuss the effects of the existing parameters on the onset of double diffusive convection numerically and graphically.

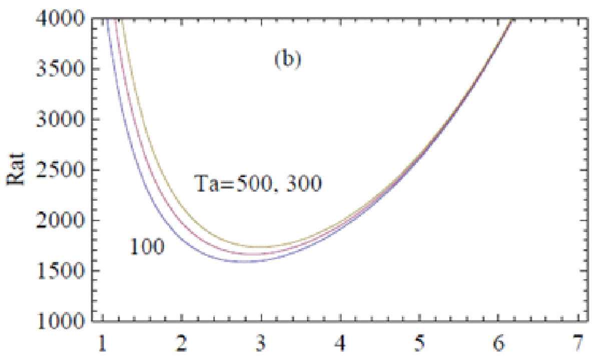

Fig.1: Variation of Stationary Rayleigh number with wave number for the different values of T_a

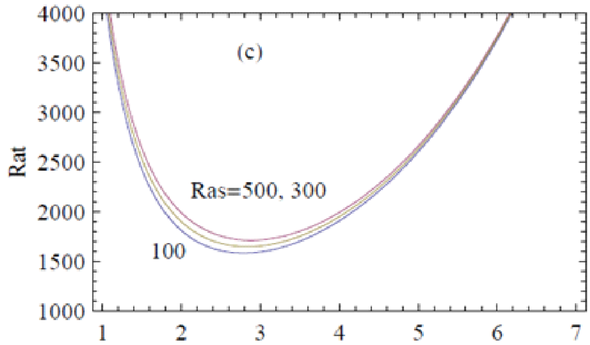

Fig.1: Variation of Stationary Rayleigh number with wave number for the different values of Ra_s

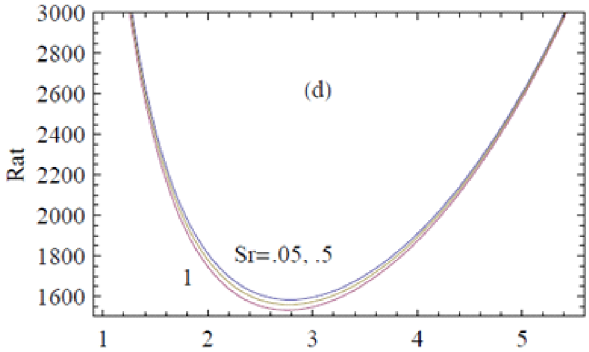

Fig.1: Variation of Stationary Rayleigh number with wave number for the different values of S_r

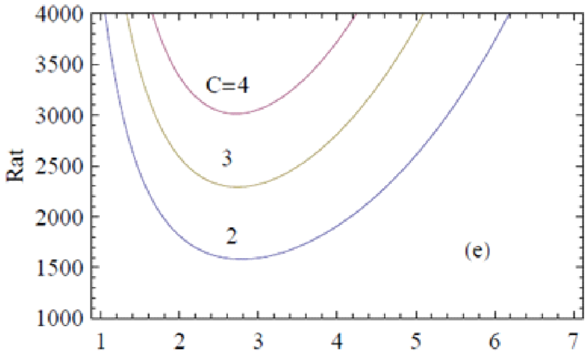

Fig.1: Variation of Stationary Rayleigh number with wave number for the different values of C

The expressions of the thermal Rayleigh number for the stationary and oscillatory modes of convection have been computed for different values of the parameters; Taylor number, couple stress parameter, solute Rayleigh number, Darcy-Prandtl number, and Soret parameter and depicted in figures.

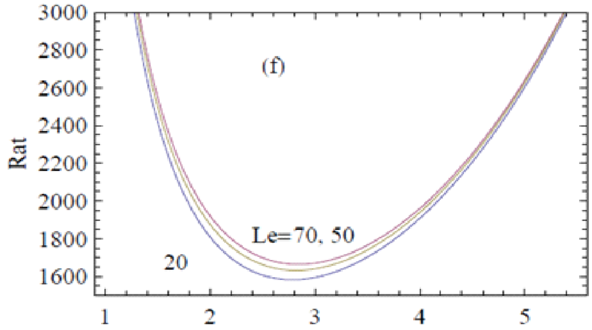

Fig.1: Variation of Stationary Rayleigh number with wave number for the different values of L_e

5.1 Linear Analysis

The marginal stability curves in the (Ra_T, a) plane for the stationary and oscillatory modes are presented for different values of various governing parameters. We fixed the values for the parameters as Ta = 100, C = 2, Ra_S = 100, L_e = 2, =.5, Pr_D = 10, S_r=.05, =.5 and R_i = 2, except the varying parameter.

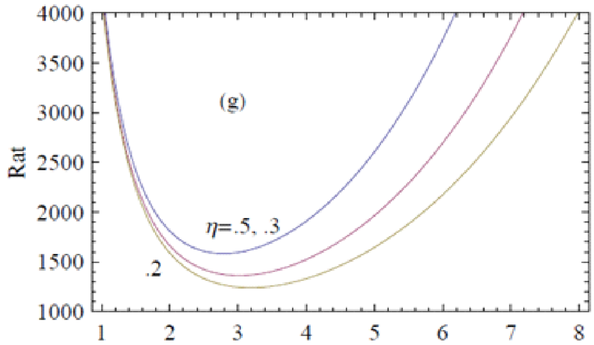

Fig.1: Variation of Stationary Rayleigh number with wave number for the different values of η

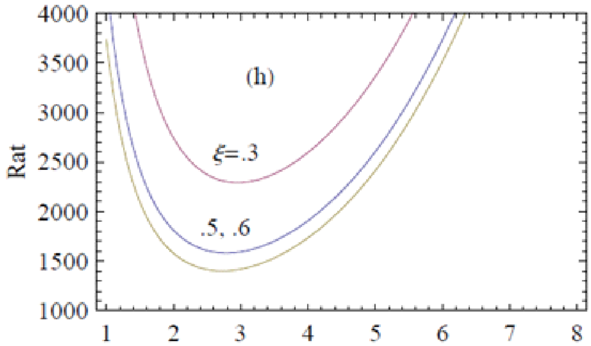

Fig.1: Variation of Stationary Rayleigh number with wave number for the different values of ξ

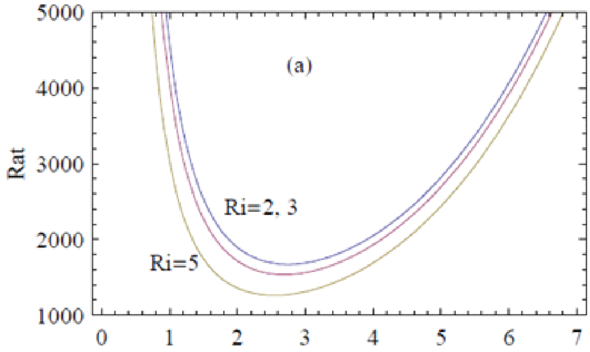

Fig.2: Variation of Oscillatory Rayleigh number with wave number for the different values of R_i

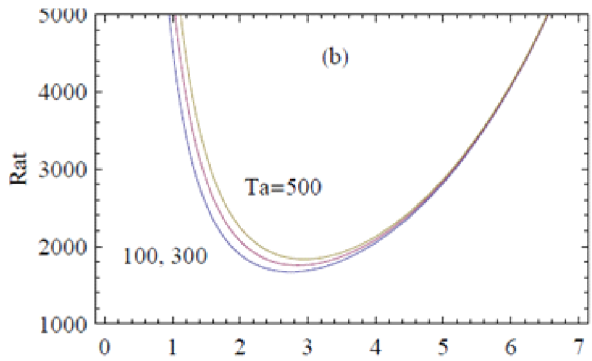

Fig.2: Variation of Oscillatory Rayleigh with wave number for the different values of T_a

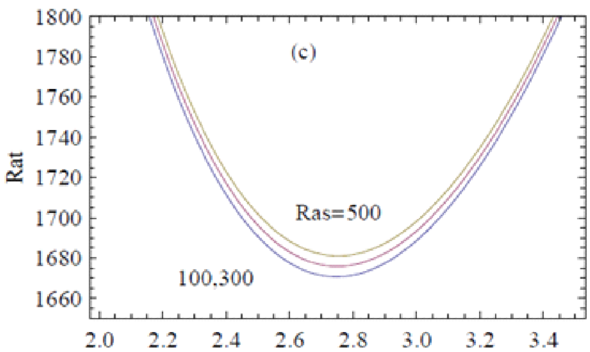

Fig.2: Variation of Oscillatory Rayleigh with wave number for the different values of Ra_s

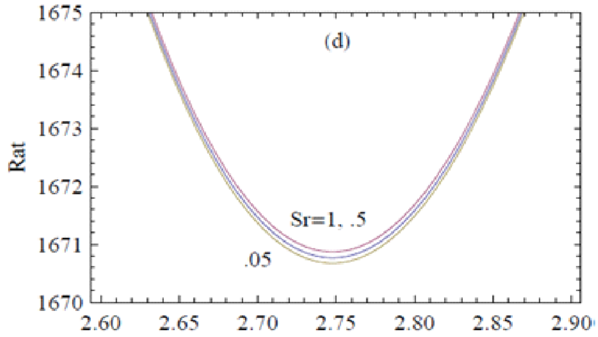

Fig.2: Variation of Oscillatory Rayleigh number with wave number for the different values of S_r

In **Fig. 1(a) and Fig. 2(a),** we observe that the stationary Rayleigh number decreases with the increase in internal Rayleigh number R_i, which indicates that the internal Rayleigh number destabilizes the system. Additionally, the increasing internal Rayleigh number decreases the minimum oscillatory Rayleigh number, which means that internal Rayleigh number has destabilizing effect on the oscillatory mode of convection as well.

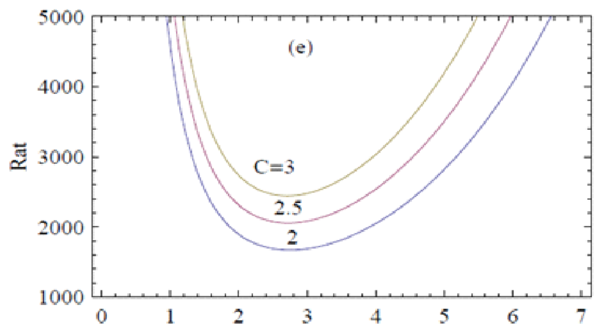

Fig.2: Variation of Oscillatory Rayleigh number with wave number for the different values of C

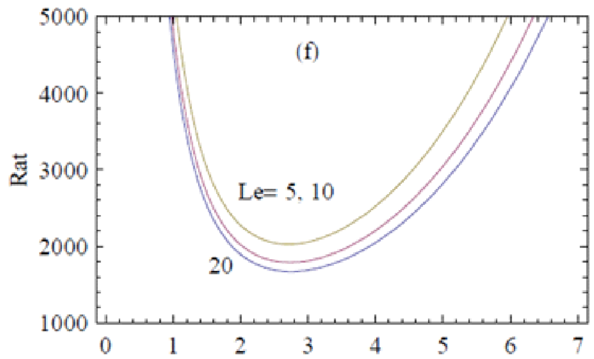

Fig. 2: Variation of Oscillatory Rayleigh number with wave number for the different values of L_e

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From **Fig. 1(b) and Fig. 2(b),** we observe that on increasing the value of Taylor number T_a, values of stationary Rayleigh number and oscillatory Rayleigh number increase, thus stabilizing the system and delaying the onset of convection. Besides, the critical wave number increases with increasing T_a.

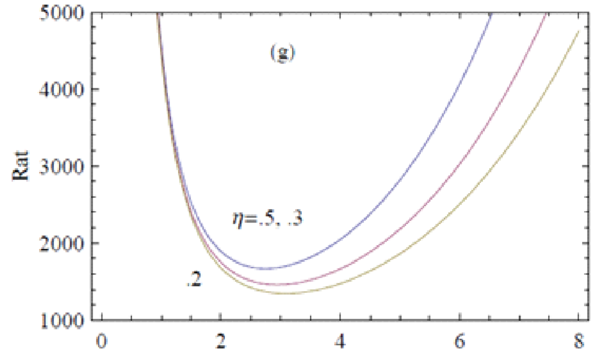

Fig. 2: Variation of Oscillatory Rayleigh number with wave number for the different values of η

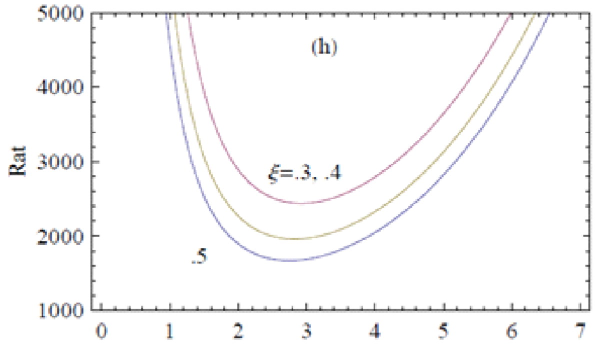

Fig. 2: Variation of Oscillatory Rayleigh number with wave number for the different values of ξ

Fig. 1(c) and Fig. 2(c) depict that the stationary and oscillatory Rayleigh number increase with an increment in solutal Rayleigh number $Ra_{S,}$ which indicates that the effect of solutal Rayleigh number is to enhance the stability of the system. Therefore, the solutal Rayleigh number Ra_S has a stabilizing effect on the system.

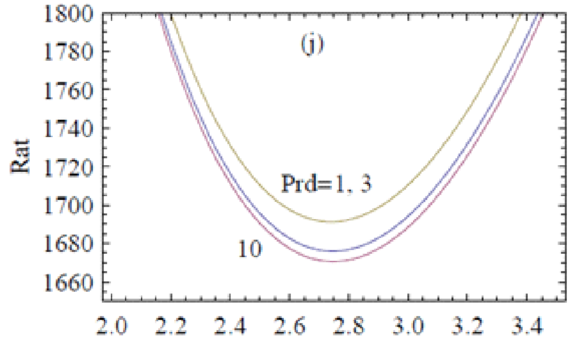

Fig.2: Variation of Oscillatory Rayleigh number with wave number for the different values of Pr_D

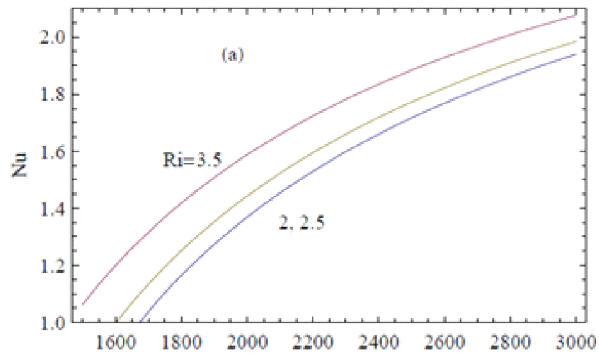

Fig.3: Variation of Nassult number with Rayleigh number for the different values of R_i

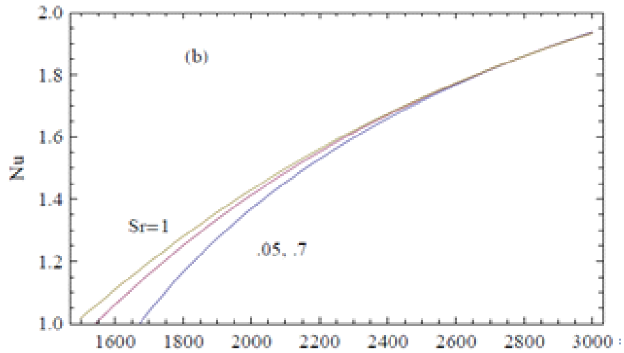

Fig.3: Variation of Nassult number with Rayleigh number for the different values of S_r

Fig. 1(d) and Fig. 2(d) show that increasing the value of Soret parameter S_r is to decrease the stationary Rayleigh number. So it has a destabilizing effect on the stationary convection. On the other hand, the oscillatory Rayleigh number increases with increasing Soret parameter, which means that the Soret parameter S_r has a stabilizing effect of the system.

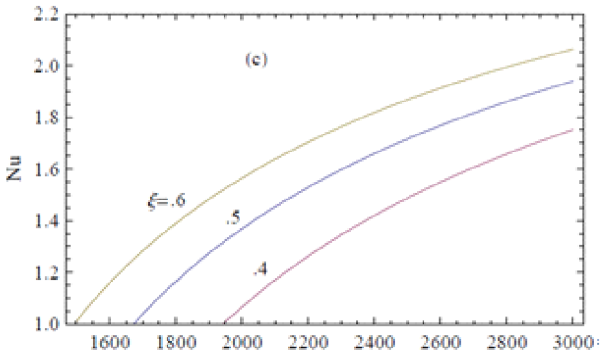

Fig.3: Variation of Nassult number with Rayleigh number for the different values of ξ

From **Fig. 1(e), (g) and Fig. 2(e), (g)**, it can be found that the increase in the value of couple stress parameter C and thermal anisotropic parameter are to increase the stationary and oscillatory Rayleigh number. Thus the system has stabilizing effect for both the modes.

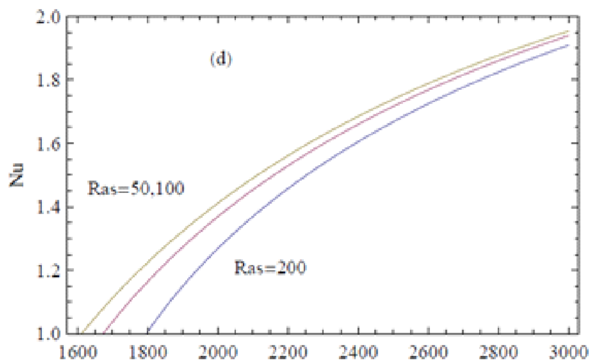

Fig. 3: Variation of Nassult number with Rayleigh number for the different values of Ra_s

We find from **Fig. 1(f) and Fig. 2(f)** that the effect of increasing the value of Lewis number L_e is to increase stationary Rayleigh number, which indicates that the Lewis number stabilizes the stationary mode of convection, while oscillatory Rayleigh number has opposite effect.

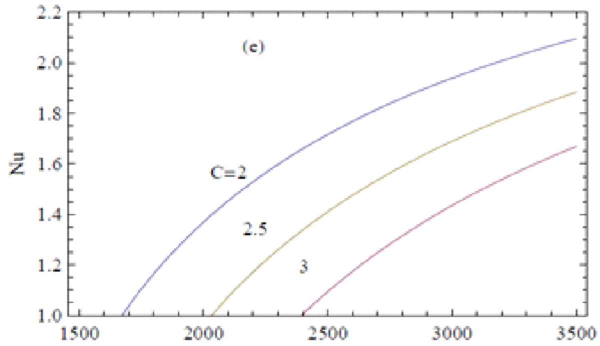

Fig.3: Variation of Nassult number with Rayleigh number for the different values of C

In **Fig. 1(h) and Fig. 2(h),** we found that increasing the value of mechanical anisotropic parameter, stationary Rayleigh number and oscillatory Rayleigh number decrease, thus is to destabilize the system. From **Fig. 2(i),** we observe that increasing the value of the Darcy-Prandtl number Pr_D decreases the oscillatory Rayleigh number, indicating that the Darcy-Prandtl number is to destabilizing the onset of oscillatory convection.

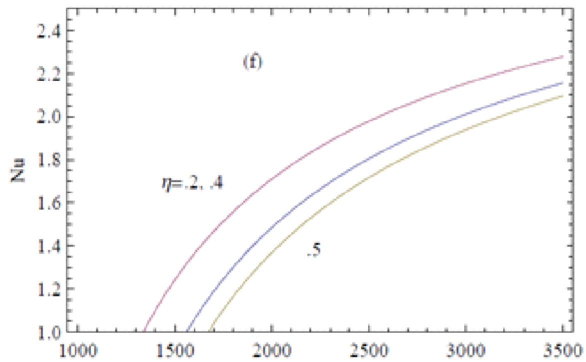

Fig.3: Variation of Nassult number with Rayleigh number for the different values of η

5.2 Nonlinear Analysis

The effect of various parameters on the rate of heat and mass transfer are shown in Fig. 3 and Fig. 4. **Fig. 3(a) and Fig. 4(a)** Show that an increase in the value of the internal Rayleigh number R_i increases both the rate of heat and mass transfer.

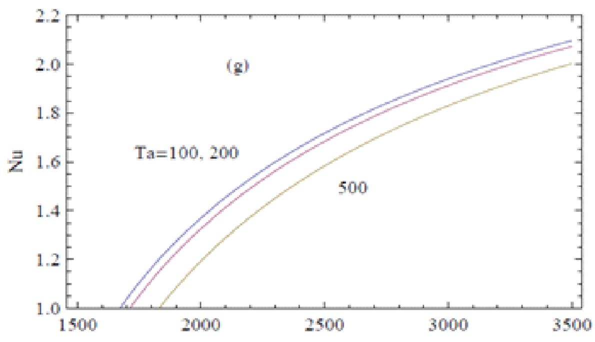

Fig.3: Variation of Nassult number with Rayleigh number for the different values of T_a

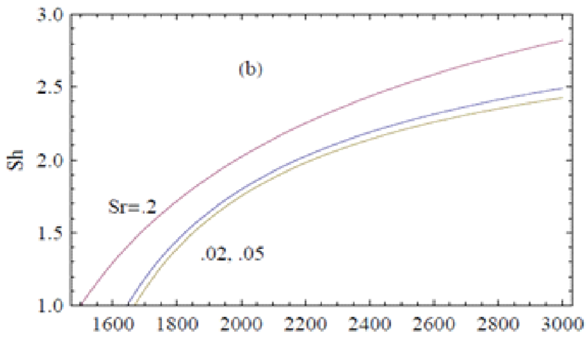

Fig. 4: Variation of Sherewood number with Rayleigh number for the different values of S_r

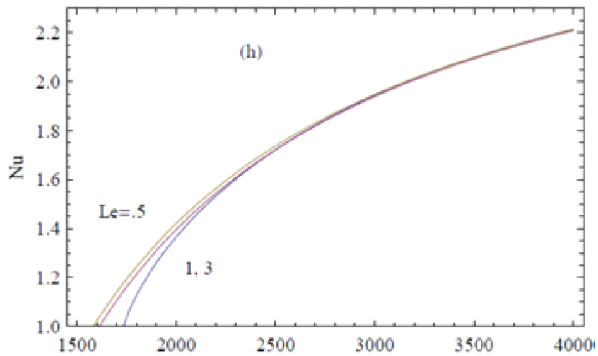

Fig.3: Variation of Nassult number with Rayleigh number for the different values of L_e

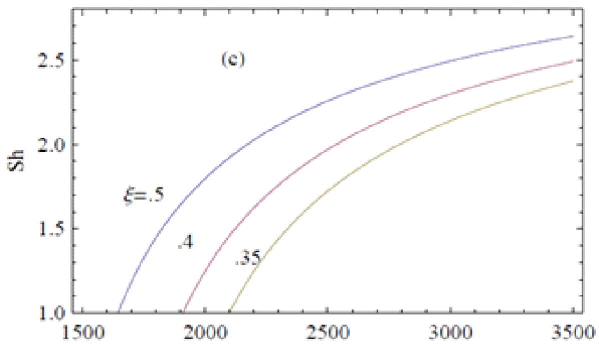

Fig.4: Variation of Sherewood number with Rayleigh number for the different values of ξ

Fig. 3(b), 3(c)-4(b),4(c) depict that the Nusselt number N_u and Sherwood number S_h increase with increasing Soret parameter S_r and mechanical anisotropic parameter, which indicates the effect of mechanical anisotropic parameter and Soret parameter S_r is to increase the rate of heat and mass transfer.

In **Fig.s 3(d),3(f)-4(d),4(g)**, it can be seen that both the rates of heat and mass transfer decrease with the increasing solute Rayleigh number Ra_S and thermal anisotropic parameter. **Fig. 3(e), 3(g) and Fig. 4(e), 4(h)**, we note that increasing the value of couple stress parameter C and Taylor number T_a are to decrease the Nusselt number N_u and Sherwood number S_h, thus reducing the heat and mass transfer.

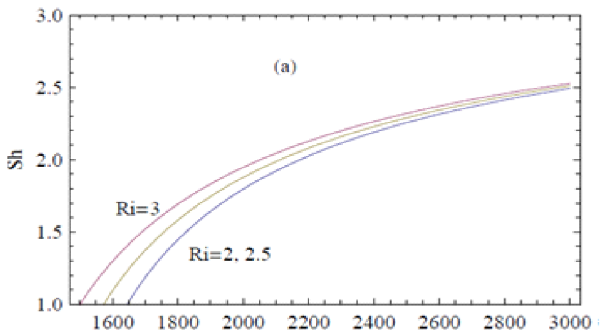

Fig. 4: Variation of Sherewood number with Rayleigh number for the different values of R_i

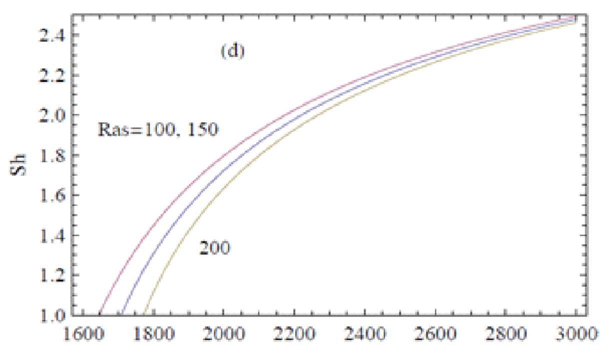

Fig. 4: Variation of Sherewood number with Rayleigh number for the different values of Ra_s

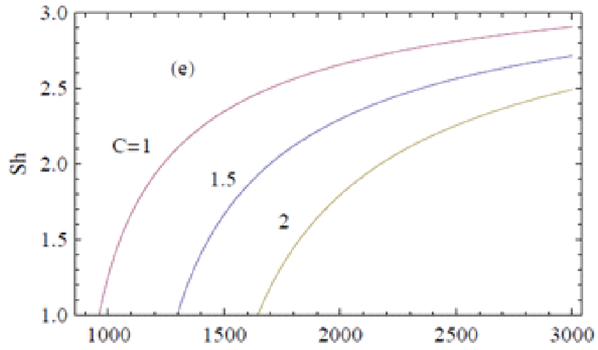

Fig. 4: Variation of Sherewood number with Rayleigh number for the different values of C

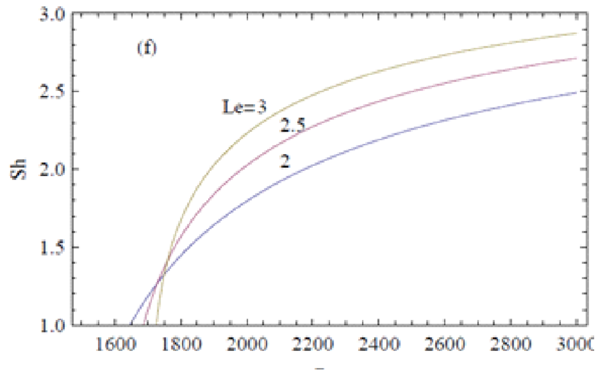

Fig.4: Variation of Sherewood number with Rayleigh number for the different values of L_e

In **Fig. 3(h)and Fig. 4(f)**, it is shown that an increase in the value of Lewis number L_e decreases the value of Nusselt number N_u and increases the value of Sherwood number S_h, thus the effect of Lewis number L_e has a stabilizing effect on heat transfer.

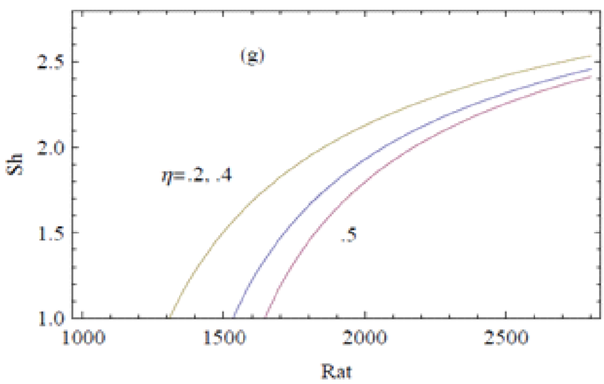

Fig.4: Variation of Sherewood number with Rayleigh number for the different values of η

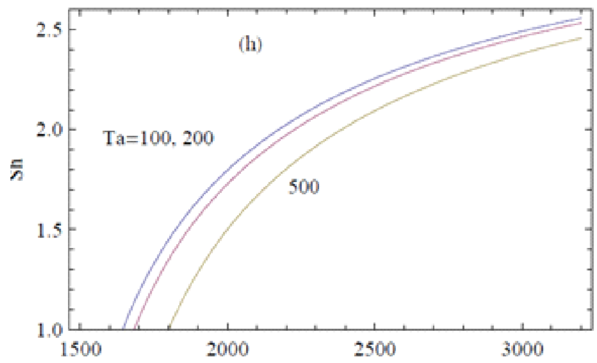

Fig.4: Variation of Sherewood number with Rayleigh number for the different values of T_a

6. CONCLUSIONS

In this paper, Soret and internal heating effect on double diffusive convection in a couple stress fluid saturated rotating anisotropic porous layer, heated and salted from below, is investigated. The problem has been solved analytically, performing linear and nonlinear analyses. The linear analysis is done using normal mode technique. Following conclusions are drawn:

1) The Taylor number T_a, Couple stress fluid C, solute Rayleigh number Ra_S and thermal anisotropic parameter has a stabilizing effect on both stationary and oscillatory modes of convection.
2) The internal heat parameter R_i , mechanical anisotropic parameter destabilize the stationary and oscillatory system.
3) The effect of Lewis number L_e has a stabilizing effect on the stationary and opposite effect has oscillatory convection.
4) The Soret parameter S_r has a stabilizing effect on the oscillatory and opposite effect has stationary convection.
5) The Darcy-Prandtl number Pr_D has a destabilize effect in case of oscillatory convection.
6) The Increasing the value of internal Rayleigh number R_i ,Soret number S_r and mechanical anisotropic parameter then increase the value of Nusselt number N_u i.e. increased heat transfer

but increasing the value of Taylor number T_a, solute Rayleigh number Ra_S, Lewis number L_e, thermal anisotropic parameter and couple stress parameter C decrease the value of Nusselt number N_u.

7) Mass transfer increased with increasing the value of Soret number S_r and mechanical anisotropic parameter, Internal Rayleigh number R_i, Lewis number L_e i.e. stable and decrease with thermal anisotropic parameter, Taylor number T_a ,and couple stress parameter C, solute Rayleigh number Ra_S i.e. destable.

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