A Bifuzzy Multi Criteria Decision Making Method for Selection of Facility Location

Shashank Chaube[1] and S. B. Singh[2]

[1,2]*Department of Mathematics, Statistics and Computer Science,*
G.B. Pant University of Agriculture and Technology, Pantnagar, Uttarakhand, India

Email: chaube.shshank@gmail.com, drsurajbsingh@yahoo.com

Abstract: The facility location selection is one of the important activities in planning of strategy in almost all of private and public industries, is a multi-criteria decision making problem which includes both quantitative and qualitative criteria. In real life situations it is very difficult to have accurate and complete information for facility location therefore traditional methods for facility location selection cannot be effectively handled. This paper proposes the integration of bifuzzy preference relation to obtain weights of criteria. Bifuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method have been proposed to rank the alternatives for dealing with incomplete/ inaccurate information on selecting the most required facility location. To illustrate the proposed method, a real life application is taken.

Keywords: AHP, Bifuzzy set, Bifuzzy Preference relation, Bifuzzy TOPSIS Method, Facility Location, Multi Criteria Decision Making.

1. INTRODUCTION

Facility location selection has a great impact on output of operating and management activities in companies. A poor choice of location might result in unnecessary transportation costs, a lack of qualified labour, lost of competitive advantage, insufficient supplies of raw materials, or some similar conditions that would be detrimental to operations. On the other hand, a good choice of location might result in some advantages such as decrease in transportation cost, maximizing the usage of resources, higher logistic performance and efficiency in operations for companies. There are large numbers of methods that have been developed for the facility location selection including MCDM (Multi Criteria Decision Making), MODM (Multi Objective Decision Making), MADM (Multi Attribute Decision Making) and group decision making. MCDM problem is the process of finding the best alternative from all feasible alternatives after qualitative or quantitative assessment of a finite set of interdependent or independent criteria. Desirable alternative can be chosen by providing preference information in terms of exact numerical value or interval. However, preference information in real life situation can be assessed in a qualitative way with vague or imprecise knowledge. In such cases, ambiguity caused by vague or imprecise preference information is a big challenge for decision makers. This fact was a great motivation for researchers to extended MCDM techniques in fuzzy environment. Technique for order preference by similarity to an ideal solution (TOPSIS), one of the most known classical MCDM methods was developed by Hwang and Yoon [3] is based upon the concept that the chosen alternative should be the closest from the positive ideal solution and the farthest from the negative ideal solution.

This paper proposes a bifuzzy multi-criteria decision making method (BFMCDM) with the TOPSIS method for selecting facility location. Bifuzzy sets introduced by Zamali et. al.[8]. In this set the membership and non-membership are not compliments to each other and their sum can also be greater than one but cannot be more than two. So we can say that conflicting bifuzzy sets (CBFS) are an extension of intuitionistic fuzzy set and preference relations based on these sets are proposed by Naim et.al.[4].

The rest of this paper is organized as follows. In the next section, basic definitions are given. Section 3 gives a detailed description of the proposed method. A practical application is given to illustrate the application of the proposed method in Section 4. Finally, conclusions of the paper are presented.

2. PRELIMINARIES

In this section basic definitions of fuzzy set, intuitionistic fuzzy set, bifuzzy set, conflicting bifuzzy set and its properties are presented. We also described different type of preference relations as follows:

Definition2.1. [7]: A fuzzy set A in the Universe of discourse X is characterized by membership function $\mu_A : X \to [0,1]$. A fuzzy set A is represented by following order pair

$$A = \{(x, u_A(x)) : \forall x \in X\} \tag{1}$$

where u_A is the grade of membership of element x in the set A.

Definition 2.2. [1]: An intuitionistic fuzzy set I on a universe X is defined as an object of the following form $I = \{\langle x, u_I(x), \nu_I(x)\rangle : \forall x \in X\}$ (2)

where the functions $\mu_I(x) : X \to [0,1]$ and $\nu_I(x) : X \to [0,1]$ represent the degree of membership and degree of non-membership of an element $x \in I \subset X$ respectively.

$\pi_I(x) = 1 - u_I(x) - \nu_I(x)$ is called degree of uncertainty or hesitation of intuitionistic fuzzy set I in X, with the condition $0 \le u_I(x) + \nu_I(x) \le 1$

Definition 2.3. [8]: Let a set X is fixed. A conflicting bifuzzy set A of X is defined :

$$A = \{\langle x, u_A(x), \nu_A(x)\rangle : \forall x \in X\} \tag{3}$$

where the functions represents the degree of positive x with respect to A and $x \in X \to \mu_A(x) \in [0,1]$, with the new condition $0 < u_I(x) + \nu_I(x) \le 1 + \xi < 2$ and all $\xi \ge 2$ by replacing the intuitionistic condition and the functions $\nu_A(x) : X \to [0,1]$ represent the degree of negative x with respect to A and $x \in X \to \nu_A(x) \in [0,1]$.

Conflicting bifuzzy sets can only be considered in certain cases when it is out of intuitionistic condition.

Definition 2.4. A fuzzy preference relation R on the set X is represented by a complementary matrix

$R = (r_{ij})_{n \times n} \subset X \times X$ for all $i, j = 1, 2, \ldots.. n$.

Definition 2.5.[6]: An intuitionistic preference relation B on a set X is represented by a matrix $B = (b_{ij})_{n \times n} \subset X \times X$ with $b_{ij} = \langle (x_i, x_j), \mu(x_i, x_j), \nu(x_i, x_j)\rangle$ for all $i, j = 1, 2, \ldots., n$. For convenience, we let $b_{ij} = (\mu_{ij}, \nu_{ij})$, for all $i, j = 1, 2, \ldots\ldots, n,$ where b_{ij} is an intuitionistic fuzzy value, composed by the certain degree μ_{ij} to which x_i is preferred to x_j and certain degree ν_{ij} to which x_i is non-preferred to x_i ,and $\pi_{ij}(x) = 1 - u_{ij}(x) - v_{ij}(x)$ is interpreted as the uncertainty degree to which x_i is preferred to x_j and $\pi_{ij}(x) = 1 - u_{ij}(x) - v_{ij}(x)$ is interpreted as the uncertainty degree to which x_i is preferred to x_j.

Definition 2.6. Let $A = \{a_1, a_2, a_3, \ldots, a_n\}$ be a finite set of alternatives and $B = \{b_1, b_2, b_3, \ldots, b_n\}$ the set of decision makers. X is a matrix of conflicting bifuzzy preference relation represented by $X = (x_{ij})_{n\times n} \subset A \times A$ for all $x_{ij} = \langle (a_i, a_j), \mu(a_i, a_j), \nu(a_i, a_j) \rangle$ for all $i, j = 1, 2, \ldots\ldots n$, where x_{ij} is a conflicting bifuzzy value, composed by the certainty degree μ_{ij} to which a_i is positively preferred to a_j and certainty degree ν_{ij} to which x_i is negatively preferred to a_j, and $0 < u_A(a) + \nu_A(a) < 2$.

A conflicting bifuzzy preference relation P is a bifuzzy subset of $A\times A$ which characterized by the following membership function:

$$\mu_{ij}(A_i, A_j) = \begin{cases} 1, & \text{if } A_i \text{ is positive definitely preferred to } A_j \\ c \in (0.5,1), & \text{if } A_i \text{ is povitive slightly preferred to } A_j \\ 0.5, & \text{if there is no preference (indifference)} \\ d \in (0.5,1), & \text{if } A_j \text{ is positive slightly preferred to } A_i \\ 0, & \text{if } A_j \text{ is positive definitely preferred to } A_i \end{cases}$$

and

$$\nu_{ij}(A_i, A_j) = \begin{cases} 1, & \text{if } A_i \text{ is negative definitely preferred to } A_j \\ c \in (0.5,1), & \text{if } A_i \text{ is negative slightly preferred to } A_j \\ 0.5, & \text{if there is no preference (indifference)} \\ d \in (0.5,1), & \text{if } A_j \text{ is negative slightly preferred to } A_i \\ 0, & \text{if } A_j \text{ is negative definitely preferred to } A_i \end{cases}$$

3. A BIFUZZY MULTI CRITERIA DECISION MAKING METHOD

In this section, the TOPSIS method is extended to Bifuzzy environment, which is a very suitable for solving decision-making problems.

Let $A = \{A_1, A_2, \ldots\ldots A_m\}$ be a set of alternatives and $C = \{C_1, C_2, \ldots\ldots C_n\}$ be a set of criteria. Bifuzzy TOPSIS method consists of the following steps:

Step 1: Construct a bifuzzy preference relation matrix. Let $B = \left(\tilde{b}_{ij}\right)_{n\times n}$ be a bifuzzy preference matrix.

$$B = \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \cdots & \tilde{b}_{1n} \\ \tilde{b}_{21} & \tilde{b}_{22} & \cdots & \tilde{b}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{b}_{m1} & \tilde{b}_{m2} & \cdots & \tilde{b}_{mn} \end{bmatrix}$$

where $\tilde{b}_{ij} = (\mu_{ij}, \nu_{ij})$ (i=1,2,…n; j=1, 2,…n) and denote the membership and non membership degree of the alternatives x_i over x_j respectively.

Step 2: Determine the weights of criteria by AHP (Analytical Hierarchy Process).

The pair-wise comparison method and the hierarchical model were developed in 1980 by T. L. Saaty [5] in the context of the AHP. AHP is an approach for decision making that involves structuring multiple choice criteria into a hierarchy, assessing the relative importance of these criteria, comparing alternatives for each criterion and determining an overall ranking of the alternatives. AHP helps to capture both subjective and objective evaluation measures, providing a useful mechanism for checking the consistency of the evaluation measures and alternatives suggested by the team thus reducing bias in decision making. The steps for implementing the AHP process for the criterion are as follows:

Step 2.1: Perform Pair-wise Comparison (Saaty nine-point preference scale is adopted for constructing the pair-wise comparison matrix).

Table 1. SATTY'S NINE POINT PREFERENCE SCALE

Scale	**Compare Factor of *i* and j**
1	Equally Important
3	Weakly Important
5	Strongly Important
7	Very Strongly Important
9	Extremely Important
2, 4, 6, 8	Intermediate value between adjacent

Step 2.2: Normalize the raw score by Geometric Mean as given below:

$$w_i = \frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}} \qquad i, \quad j \quad = \quad 1,2,\ldots n$$

(4)

Step 3: Construct weighted bifuzzy decision matrix. After the weights of criteria are determined, the weighted bifuzzy decision matrix is constructed.

Step 4: Obtain bifuzzy positive-ideal solution (BFPIS) and bifuzzy negative-ideal solution (BFNIS).

In general, the evaluation criteria can be categorized into two kinds, benefit and cost. Let G be a collection of benefit criteria and B be a collection of cost criteria. According to bifuzzy theory and the principle of classical TOPSIS method, BFPIS and BFNIS can be defined as:

$$A^{+} = [\{C_j, \left\langle (\max_i \mu_{ij}(C_j) / j \in G), (\min_i \mu_{ij}(C_j) / j \in B) \right\rangle,$$
$$\left\langle (\min_i \nu_{ij}(C_j) / j \in G), (\max_i \nu_{ij}(C_j) / j \in B) \right\rangle\} \mid i \in m]$$

(5)

$$A^{-} = [\{C_j, \langle(\min_i \mu_{ij}(C_j) / j \in G), (\max_i \mu_{ij}(C_j) / j \in B)\rangle,$$
$$\langle(\max_i \nu_{ij}(C_j) / j \in G), (\max_i \nu_{ij}(C_j) / j \in B)\rangle\} \mid i \in m]$$
(6)

Step 5: Calculate the distance measures of each alternative A_i from BFPIS and BFNIS.
We use intuitionistic separation measures characterized by maximum to help in determining the ranking of all alternatives.

$$S_i = \sum_{i=1}^{n} \max\{|\mu_A(x_i) - \mu_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}$$
(7)

Step 6: Calculate the relative closeness coefficient (CC) of each alternative and rank the preference order of all alternatives. The relative closeness coefficient (cc) of each alternative with respect to the bifuzzy ideal solutions is calculated as:

$$C_j = \frac{S^{-}}{S^{+} + S^{-}} \quad \text{where} \quad 0 < c_j < 1 \quad j=1,2,... \quad m.$$

(8) The larger value of C_j indicates that an alternative is closer to BFPIS and farther from BFNIS simultaneously. Therefore, the ranking order of all the alternatives can be determined according to the descending order of CC values. The most preferred alternative is the one with the highest value.

4. NUMERICAL EXAMPLE

A manufacturing company has to select a location for building new plant. There are four candidates (A_1, A_2, A_3, A_4) chosen for further evaluation. In order to evaluate candidate locations, expansion possibility (C_1), availability of acquirement material (C_2), community considerations (C_3), distance to market (C_4) are considered as evaluation factors.

Step 1: Construct a bifuzzy preference relation matrix.

Table 2. BIFUZZY PREFERENCE RELATION MATRIX

Alternatives	C_1	C_2	C_3	C_4
a_1	(0.50,0.50)	(0.60,0.50)	(0.70,0.40)	(0.50,0.60)
a_2	(0.10,0.90)	(0.50,0.50)	(0.40,0.80)	(0.90,0.20)
a_3	(0.60,0.50)	(0.80,0.30)	(0.50,0.50)	(0.90,0.20)
a_4	(0.80,0.30)	(0.70,0.10)	(0.80,0.40)	(0.50,0.50)

Step 2: Determine the weights of criteria by AHP.

Table 3. PAIRWISE COMPARISON MATRIX

	C_1	C_2	C_3	C_4
C_1	1	3	5	1/7
C_2	1/3	1	9	5
C_3	1/5	1/9	1	3
C_4	7	1/5	1/3	1

Using equation (4) one can get the weights of criteria as

$W_1 = 0.27 \quad W_2 = 0.44 \quad W_3 = 0.11 \quad W_4 = 0.18$

Step 3: Therefore, the weighted bifuzzy decision matrix is commuted in Table 4 by multiplying bifuzzy values with weights of the criteria.

Table 4. WEIGHTED BIFUZZY DECISION MATRIX

	c_1	c_2	c_3	c_4
a_1	(0.135, 0.135)	(0.264, 0.22)	(0.077, 0.044)	(0.09, 0.108)
a_2	(0.027, 0.243)	(0.22, 0.22)	(0.044, 0.088)	(0.162, 0.036)
a_3	(0.162, 0.135)	(0.352, 0.132)	(0.055, 0.055)	(0.162, 0.036)
a_4	(0.216, 0.081)	(0.308, 0.044)	(0.088, 0.055)	(0.090, 0.090)

Step 4: After the weighted bifuzzy decision matrix is determined, the bifuzzy positive-ideal solution and bifuzzy negative-ideal solution were obtained as follows:

$\mathbf{A}^+ = \{(0.216,0.81)\ \ (0.352,0.44)\ \ (0.088,0.044)\ \ (0.162,0.09)\ \}$

$\mathbf{A}^- = \{(0.027,0.243)\ \ (0.22,0.22)\ \ (0.044,0.088)\ \ (0.09,0.108)\}$

Step 5: Then, separation of each alternative from the positive-ideal solution and negative-ideal solution is given as Table 5:

Table 5. Separation measures

	S_i^+	S_i^-
A_1	0.252	0.196
A_2	0.419	0.072
A_3	0.274	0.570
A_4	0.127	0.340

Step 6: The relative closeness coefficients are given in Table 6:

Table 6. RELATIVE CLOSENESS COEFFICIENTS

	C_i
A_1	0.5625
A_2	0.1466
A_3	0.6753
A_4	0.7280

Step 7: Finally, ranking of the preference order has been determined and then four alternatives have been ranked according to descending order of relative closeness coefficients values. $A_4 > A_3 > A_1 > A_2$ **.** Hence amongst the four, best location is A_4.

5. CONCLUSION

The paper has been designed to perform a case study in order to show how the BFMCDM can be used in facility location selection problem. In the multi-criteria decision making (MCDM) field, there are several crisp, fuzzy or intuitionistic fuzzy ranking methods that only provide decision makers (DMs) with a ranking order of alternatives, but in bifuzzy aspect it considers both negative and positive aspect. In this paper both ranking and information is provided by Bifuzzy TOPSIS and AHP method. An example is also taken to implement the proposed multi-criteria decision making process to rank the alternatives.

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