Free Convective MHD Flow through Porous Medium

Madan Lal[1] and Yantish Dev Jha[2]

[1,2]*Department of Applied Maths,M.J.P. Rohilkhand University, Bareilly, 243006. (India)*

Email: yantishdev.jha@gmail.com

Abstract: In the present paper, the effect of equilibrium temperature gradient and magnetic field on the rate of heat transfer, velocity and temperature has been studied. Numerical calculations for the rate of heat transfer, velocity and temperature are obtained and shown graphically. The highly porous medium which is bounded by an infinite vertical non conducting plane surface has been considered and two dimensional free convective MHD flow of a stratified viscous fluid has discussed.

Key words: Free Convective effect, MHD flow, Viscous fluid, Magnetic field, Porous medium.

1. INTRODUCTION

The unsteady and steady transmission of a homogeneous viscous fluid by means of a porous medium and is surrounded by a vertical plate with varying constant temperature has studied by 'Raptis *et al.*' and 'Raptis'. 'Sharma' suggested the flow for free convective effect with ordinary viscous fluid to infinite vertical porous plate having constant suction and heat flux. 'Archary *et al.*' investigated the impact of magnetic field effect on the mass transfer flow for free convection through porous medium having constant suction and heat. 'Noushima *et al.*' investigate unsteady free convective MHD flow for walter's memory with constant suction and heat sink. 'Magyari *et al.*' studied the analytical study for unsteady free convection for porous media. 'Soundalgekhar' studied the various aspect for horizontal magnetic field and variable suction on the free convection flow with vertical porous plate and denoted the different parameters and flow of mercury and ionized air. 'Turchuk and Shildovskill' studied the equations of motion for the fluid having viscous micro layers, its relation with viscosity, thermal conductivity and diffusion, deviation of field variable from hydrostatic value and the effect of equilibrium stratification through equilibrium density gradient. The unsteady free convective flow of a thermally stratified viscous fluid through porous medium was studied 'Mondal and Chaudhary'.

The present paper is the study of unsteady free convective MHD flow in two dimensional stratified viscous conducting fluid through highly porous medium having infinite vertical non-conducting plane under the effect of a homogeneous transverse magnetic field. . It is observed that the temperature of the surface, which has oscillating behavior with respect to the time, varies vertically. The energy transport is effected by force distributed is due to density variation for convection for free flow. The flow in a especially porous medium is the main aspect for the study due to many application in industrial problem such as petroleum, nuclear and chemical industries. The magnetic effects are extremely used in the power generation.

2. FORMULATION AND SOLUTION OF THE PROBLEM:

Unsteady two dimensional flow of a viscous conducting and thermally stratified fluid by highly porous medium surrounded by an unlimited non conducting vertical plane, which is moving vertically having a constant velocity V has considered. We have also consider the x′- axis, which is vertically upward along with the plane, y' axis is normal to the plane $B_0(=q_0H_0)$ shows uniform magnetic field strength, this value represent the normal action of the plane. The relationship between the temperature and pressure of a fluid have the following inter-relationship $\theta' = \theta_e + T'$, $\sigma' = \sigma_e - \alpha\sigma_0 T'$ and p' = $p_e + \bar{p}$ .The temperature θ_e at equilibrium, density σ_e and pressure of a fluid p_e have the following relationship $\theta_e = \theta_0 + A'_F x'$, $\sigma_e = \sigma_0(1-\alpha\theta_0)$ and $\frac{dp_e}{dx'} = -\sigma_e g$ where T' and p indicates the temperature deviation and pressure deviation respectively, equilibrium temperature gradient A'_θ for fluid is the density is represented by σ_0, where θ_0 represents the temperature, coefficient of volume expansion is denoted by α, where g is acceleration due to gravity.

The plate temperature is considered as follows:

$$\theta'_\omega = \theta_e + \theta_w\left(1+\epsilon\, e^{i\omega' t'}\right)$$

In the above equation $\theta_w > 0$, the time is indicated by t', which is constant, the frequency of the oscillation is ω', where as $\epsilon(<1)$ is a constant value.

Concerning T' $\overline{p}$ and the components of velocity, these are independent of x', equation of momentum and energy neglecting viscous dissipation can be represented as

$$\frac{\partial q'}{\partial t'} = \alpha g\, T' - \frac{\mu}{K' q'} + \mu\frac{\partial^2 q'}{\partial y'^2} - \frac{\rho}{\sigma} B_0^2 q' \quad (1)$$

$$\frac{\partial T'}{\partial t'} = -A'_\theta q' + \frac{K}{\sigma_0 C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\rho}{\sigma} B_0^2 q' \quad (2)$$

Where q' represents the velocity components along x' axis, k denotes the thermal conductivity, C_p represents the specific heat at constant pressure, K' represents permeability of the medium and μ denotes the kinematic coefficient of viscosity.

Now we consider the non dimensional variables as follows:

$$y = \frac{y' V}{\mu}, \quad q = \frac{q'}{V}, \quad T = \frac{T'}{\theta_w}, \quad \omega = \frac{\mu\,\omega'}{V^2} \text{ and } t = \frac{t' V^2}{\mu}$$

The above concerning equations can be represented as

$$\frac{\partial^2 q}{\partial y^2} - \frac{\partial q}{\partial t} - mq = -GT, \quad (3)$$

$$\frac{\partial^2 T}{\partial y^2} - P_r\frac{\partial T}{\partial t} = A_\theta P_r q - \mathcal{E} M P_r q^2, \tag{4}$$

Where Grashoff number = $\frac{\alpha g \theta_w \mu}{V^3}$, Prandtl number $= \frac{\mu C_p \sigma_0}{k}$, Permeability parameter = $\frac{V^2 K'}{\mu^2}$,

Equilibrium temperature gradient parameter $= \frac{\mu A'_\theta}{\theta_w V}$, Hartmann number $= \frac{\mu \rho B_0^2}{V^2 \sigma}$ and Eckert number $= \frac{V^2}{\theta_w}$ <<1 are denoted by G, P_r, K, A_θ, M and E respectively .

The boundary conditions are considered as follows:

$q = 1$, $T = 1 + \epsilon e^{i\omega t}$ at y = 0

$q, T \to 0$ as $y \to \infty$ (5)

In view of above boundary conditions (5) the solution of equations takes (3) and (4) the form

$$q = \sum_{r=0}^{\infty} \epsilon^r q_r(y) e^{i\omega r t},$$

$$T = \sum_{r=0}^{\infty} \epsilon^r T_r(y) e^{i\omega r t}. \tag{6}$$

If we consider the non harmonic terms and the harmonic terms then substituting equations (6) in equations (3) and (4). We get the differential equations in (q_0, T_0) and (q_1, T_1)

$$\frac{d^2 q_0}{dy^2} - m q_0 = -G T_0; \quad \frac{d^2 T_0}{dy^2} = A_\theta P_r q_0; \tag{7}$$

$$\frac{d^2 q_1}{dy^2} - (i\omega + m) q_1 = -G T_1; \quad \frac{d^2 T_1}{dy^2} - i\omega P_r T_1 = A_\theta P_r q_1; \tag{8}$$

In reduced boundary conditions, we can find another equation (9)

$q_0 = 1, q_1 = 1, T_0 = 1, T_1 = 1$ at y = 0

$q_0, q_1, T_0, T_1 \to 0,$ as $y \to \infty$ (9)

In the view of above boundary conditions the solution of equations can be obtained in the form of velocity, and temperature field as below:

Case -I

If $A_\theta < \frac{m^2}{4GP_r}$

Then $$q = \sum_{r=0}^{\infty} \epsilon^r \left(A_r e^{-\alpha_r y} + B_r e^{-\beta_r y}\right) e^{i\omega r t} \qquad (10)$$

$$T = -\frac{1}{G}\left[\sum_{r=0}^{\infty} \epsilon^r \left\{\left(\alpha_r^2 - i\omega r - m\right)A_r e^{-\alpha_r y} + \left(\beta_r^2 - i\omega r - m\right)B_r e^{-\beta_r y}\right\} e^{i\omega r t}\right] \qquad (11)$$

Case -II

If $A_\theta > \frac{m^2}{4GP_r}$

then

$$q = e^{-m_1 y}(Cosm_2 y + Sinm_2 y) + \sum_{r=1}^{\infty} \epsilon^r \left(A_r e^{-\alpha_r y} + B_r e^{-\beta_r y}\right) e^{i\omega r t} \qquad (12)$$

$$T = -\frac{e^{-m_2 y}}{G}\left[\left\{m_1^2 - m_2^2 - m - 2m_1 m_2 C\right\} Cosm_2 y + \left\{C\left(m_1^2 - m_2^2 - m\right) + 2m_1 m_2\right\} Sinm_2 y\right] - \frac{1}{G}\left[\sum_{r=1}^{\infty} \epsilon^r \left\{\left(\alpha_r^2 - i\omega r - m\right)A_r e^{-\alpha_r y} + \left(\beta_r^2 - i\omega r - m\right)B_r e^{-\beta_r y}\right\} e^{i\omega r t}\right] \qquad (13)$$

Case –III

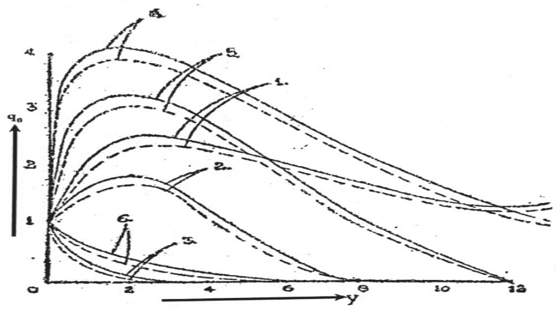
If $A_\theta = \frac{m^2}{4GP_r} = \overline{A_\theta}$ (Critical value),

Then $$q = (1 + Fy)\, e^{-\sqrt{\frac{m}{2}}y} + \sum_{r=1}^{\infty} \epsilon^r \left(A_{r+1} e^{-\alpha_{r+1} y} + B_{r+1} e^{-\beta_{r+1} y}\right) e^{i\omega r t} \qquad (14)$$

$$T = \frac{1}{G}\left[\frac{\left(F\sqrt{8K} + y\right) + (1 + MK)}{2K}\right] e^{-\sqrt{\frac{m}{2}}y} - \frac{1}{G}\left[\sum_{r=2}^{\infty} \epsilon^{r-1}\left\{\left(\alpha_r^2 - i(r-1)\omega - m\right)B_r e^{-\beta_r y}\right\} e^{i(r-1)\omega t}\right] \quad (15)$$

Where

$$(\alpha_0, \beta_0) = \left[\frac{1}{2}\left\{m \pm \sqrt{(m^2 - 4P_r G A_\theta)}\right\}\right]^{\frac{1}{2}},$$



$$(m_1, m_2) = \left[\frac{(4P_r G A_\theta K^2 \pm 1)(1 + MK)}{4K}\right]^{\frac{1}{2}},$$

$(\alpha\_1, \beta\_1) = [1/2\ \{i\omega(1 + P\_r) + m \pm \sqrt{((m^\wedge 2 - 4P\_r\, GA\_\theta - \omega^\wedge 2\ (1 - [P\_r)]^\wedge 2 + 2i\omega m(1 - P\_r)\ )\}]^\wedge(1/2)$

$(\alpha\_2, \beta\_2) = [1/2\ \{i\omega(1 + P\_r) + m \pm \sqrt{(2i\omega/M\ (1 + MK)(1 - P\_r) - \omega^\wedge 2\ (1 - [P\_r)]^\wedge 2\ )\}]^\wedge(1/2)$,

$$A_0 = -\frac{\beta_0^2 + G - m}{\alpha_0^2 - \beta_0^2}, \qquad B_0 = \frac{\alpha_0^2 + G - m}{\alpha_0^2 - \beta_0^2}, \qquad m = \frac{1}{K} + M,$$

$$C = \frac{1}{2m_1 m_2}\left[G + (m_1 m_2)^2 - m_2^2 - m\right], \quad A_1 = -B_1 = \frac{G}{\beta_1^2 - \alpha_1^2}$$

$$A_2 = -B_2 = \frac{G}{\beta_2^2 - \alpha_2^2} \text{ and } F = \left(\frac{2KG - 1}{8K}\right)(1 + MK)^{\frac{1}{2}}.$$

The velocity field can be defined as

$q(y,t) = q_0(y) + \epsilon q_r(y)\cos\omega t + \epsilon q_i(y)\sin\omega t$, here q_r and q_i ,these are real and imaginary parts of q_1. If $\omega t = \frac{\pi}{2}$, then the relation for transient velocity becomes

$$q_\theta = q_0(y) + \epsilon q_i(y)$$

The indication for the transient velocity q_θ for different values of K, G, M and A_θ with P_r = 0.71 and $\epsilon = 0.02$ are indicated in Fig. 1 and Fig. 2.

A_θ		
	M	K=1
1	0 2	0.01 0.12
2	0 2	0.088 0.792
3	0 2	0.10 0.80
4	0 2	0.01 0.25
5	0 2	0.022 0.550
6	0 2	0.05 0.60

Fig.1. Variation in distribution of transient velocity at $\epsilon = 0.02, P_r = 0.71$, G = 4, $\omega = 6$, for $M = 0$ and $M = 2$. *The values of* $\overline{A_\theta}$ for $K = 1$ *and* $K = 2$ are corresponds to curve 2 and curve 5.

Heat transfer rate at ω t= $\frac{\pi}{2}$ the plane for in terms of Nusselt number N = $-\left(\frac{d\theta}{dy}\right)_{y=0}$ are shown in table-1

Table – I

Heat transfer rate in terms of variation in Nusselt number N, for various numerical values of ω , G and A_θ (P_r = 0.71 $\epsilon = 0.02$) .

ω	G	K	A_θ	N(M=0)	N(M=2)
5	4	1	0.01 0.05 0.10	-0.146 0.029 0.129	-0.152 0.031 0.134
		2	0.01 0.05 0.10	-0.106 0.077 -0.192	-0.112 0.086 -0.198
	8	1	0.01 0.05 0.10	-0.091 0.104 0.230	-0.104 0.117 0.239

		2	0.01 0.05 0.10	-0.043 0.163 0.291	-0.048 0.172 0.301
10	4	1	0.01 0.05 0.10	-0.226 -0.065 0.045	-0.232 -0.073 0.049
		2	0.01 0.05 0.10	-0.185 0.008 0.106	-0.192 -0.009 0.011
	8	1	0.01 0.05 0.10	-0.172 0.031 0.143	-0.179 0.038 0.151
		2	0.01 0.05 0.10	-0.117 0.082 0.210	-0.119 0.088 0.217

3. DISCUSSION:

It is obvious by the figures 1 and 2 that there is very high relationship between equilibrium temperature gradient parameter (A_θ) and transient velocity (q_θ)

A_θ		
N	M	K=1
1	0 2	0.01 0.09
2	0 2	0.044 0.369
3	0 2	0.05 0.40
4	 0 2	K=2 0.01 0.12
5	0 2	0.011 0.305
6	0 2	0.012 0.38

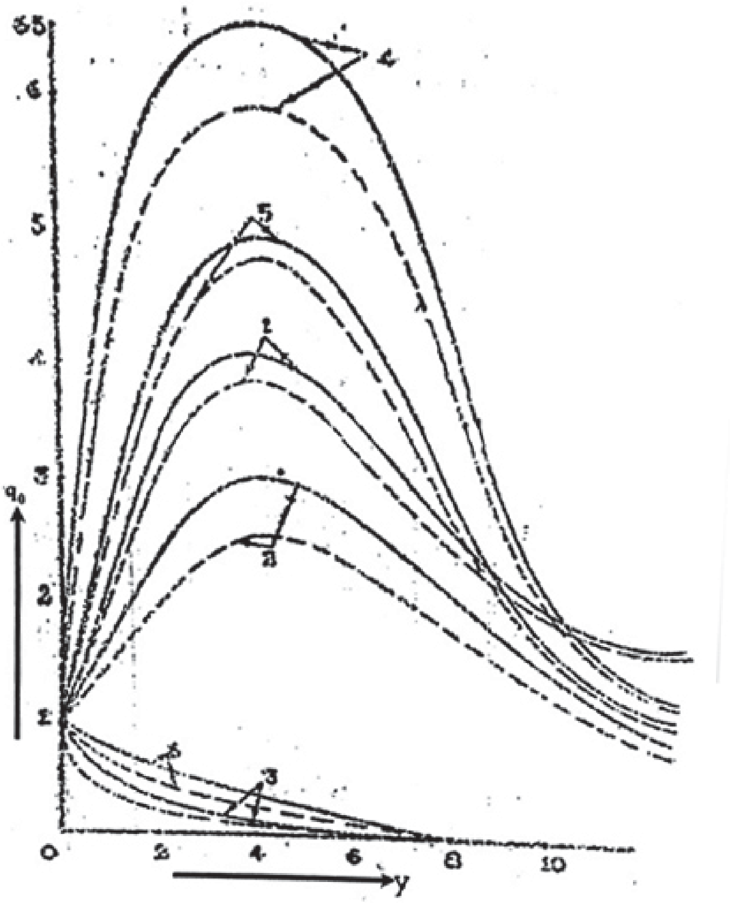


Fig. 2. Distribution of transient velocity at $\epsilon = 0.02$, P_r = 0.71, G = 8, $\omega = 6$, for M = 0 and M = 2. Curve 2 and curve 5 denoted the critical values of $\overline{A_\theta}$ *for K* = 1 *and K* = 2 .

If we increase the equilibrium temperature gradient parameter (A_θ), then the transient velocity $(q_\theta$) decrease all the time. The fluid velocity relative to the plate at the first increase with the increasing from the plane and gains the maximum value within the fluid and decrease exponentially for conditions 0< $A_\theta \leq \overline{A_\theta}$ $(\overline{A_\theta} = \textit{critical value})$. If conditions are like as $A_\theta > \overline{A_\theta}$, the relative velocity decrease. If we increase the distance of plane and the fluid flow relative to the plane condition and this is opposite to the direction of the motion of the plane, in these situations the magnetic field decreases in all. It shows that the application of magnetic field is not favorable to the direction of flow. If the medium of flow is non porous letting $K \to \infty$ as well as $\overline{A_\theta} = \frac{M^2}{4GP_r} < A_\theta$ then the velocity of fluid is always less than that of the plane.

Grashoff number shows the free convection effect, for $G > 0$ response the cooled surface formulation. When the G, K and ω increases the velocity will also increases.
In table-I Nusselt number N with reference to the heat transfer effect has explained, which shows that for small A_θ, the heat is transferred from the fluid to the plane, but there is reverse condition for higher A_θ.

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