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Relativistic Modeling of Gravitationally Collapsing Anisotropic Fluid Ball with Occurrence of Horizon

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Abstract

We here investigate a relativistic model together with some new exact class of solutions for an anisotropic radiating star using Tewari and Charan [1] solution; as a seed solution. The interior metric obeyed all the relevant physical and thermodynamic conditions and matched with Vaidya exterior metric over the boundary. The model is physically and thermodynamically sound as it corresponds to the non-negative expressions for non homogeneous fluid density, both the radial and transverse pressures and radiation flux density throughout the fluid ball. Initially the interior solutions represent a static configuration of perfect fluid which then gradually starts evolving into radiating collapse. Consequently we have obtained the expressions of mass energy, physical radius, apparent luminosity, surface redshift and surface temperature of collapsing radiating star.

1. Introduction

Gravitational collapse has many fundamental phenomenon applications in relativistic astrophysics where the formation of compact objects as white dwarfs, black holes, naked singularities, Supernovae, neutron star, and strange stars are usually found. In view of Cosmic Censorship Conjecture of Penrose [2]; nature avoid naked singularities but there are a number of counter examples available in the literature where naked singularity has more chances to be formed(for a recent review see Joshi and Malafarina [3]; and references therein). The description of the gravitational collapse was first proposed by Oppenheimer and Snyder [4]; in which they assumed a spherically symmetric fluid distribution of matter and equation of state in the form of dust with Schwarzschild exterior. Taking to account the outgoing radiation from collapsing spherical fluid, Vaidya [5]; initiated the problem of solving the relativistic field equations. Later on modified equations were proposed by Misner [6]; Lindquist et al. [7]; for an adiabatic distribution of matter.

It is an established fact that gravitational collapse is highly dissipating energy process (Herrera and Santos [8], Herrera et al. [9] Mitra [10] and references therein) which plays a dominant role in the formation and evolution of stars. Santos [11] studied the junction conditions of collapsing spherically symmetric shear-free non-adiabatic fluid with radial heat flow which was based on relativistic models suggested by Glass [12]. On the similar ground a number of stellar models (de Oliveira and Santos [13]; Bonnor et al. [14]; Banerjee et al. [15]; Maharaj and Govender [16]; Debnath et al. [17], Herrera et al.[18, 19, 20], Naidu and Govender [21], Tewari [22, 23, 24, 25, 26]; Sarwe and Tikekar [27]; Sharma and Tikekar [28]; Ivanov [29]; Govinder and Govender [30]; Maharaj et al. [31]; Pinheiro and Chan [32]; Tewari and Charan [33, 34]; Virdhadra [35]; Vaidya [36]; Herrera [37, 38], Bowers and Liang [39]; Ivanov [40, 41, 42]; Tewari, Charan and Rani [43]; Pandey and Sharma [44] and also references therein) have been reported with the impact of various factors such as shear, in homogeneity, anisotropy, electromagnetic field and various dissipative processes on the evolution.

Keeping in view of generality of solution due to Tewari and Charan [1]; we present a special solution and its detailed study, in order to construct a realistic model of collapsing radiating star together with a table of class of exact solutions, which will be fruitful for the further study. The energy momentum tensor corresponding to the fluid distribution filling the interior of the collapsing star however has been assume to be anisotropic in general .The interior space-time is matched with Vaidya exterior metric [5]; over the boundary, and the final fate of our model is formation of black hole.

2. The field equations and the junction conditions

We consider a spherically symmetric distribution of fluid undergoing dissipation in the form of heat flow bounded by a time-like spherical surface Σ. The interior space-time is described by the metric

$$ds_-^2 = -X^2(r,t)dt^2 + U(r,t)\{dr^2 + r^2(d\theta^2 + sin^2\theta d\psi^2)\} \tag{1}$$

The energy-momentum tensor for the interior matter distribution with radial heat flow is given by

$$T_{\lambda\omega} = (\epsilon + p)\eta_\lambda\eta_\omega + pg_{\lambda\omega} + (p_r - p_t)\tau_\lambda\tau_{\omega+}q_\lambda\eta_\omega + q_\omega\eta_\lambda \tag{2}$$

where ϵ the energy density of the fluid, p_r the radial pressure, p_t the tangential pressure, η_λ is four- velocity and q_λ the radial heat flow vector and τ_λ is a unit space like four vectors along the radial direction. Assuming commoving coordinates, we have $\eta^\lambda = \frac{1}{X}\delta_0{}^\lambda$ .The heat flow vector q^λ is orthogonal to velocity vector so that $q^\lambda\eta_\lambda = 0$ and takes the form $q^\lambda = q\delta_1{}^\lambda$.The exterior space-time is described by Vaidya's exterior metric [5] which represents an outgoing radial flow of radiation.

$$ds_+{}^2 = -\left(1 - \frac{2M}{R}\right)dv^2 - 2dRdv + R^2(d\theta^2 + sin^2\theta d\psi^2) \tag{3}$$

where v is the retarded time and $M(v)$ is the exterior Vaidya's [5] mass.

The junction conditions for matching two line elements (1) and (3) continuously across a spherically symmetric time-like hyper surface Σ are well known and obtained by Santos [11]

$$(rU)_\Sigma = R_\Sigma(v) \tag{4}$$

$$(p_r)_\Sigma = (qU)_\Sigma \tag{5}$$

$$m_\Sigma(r,t) = M(v) = \left\{\frac{r^3U\dot{U}^2}{2X^2} - r^2U' - \frac{r^3U'^2}{2U}\right\}_\Sigma \tag{6}$$

where m_Σ is the mass function calculated in the interior at $r = r_\Sigma$ (Chill et al. [45] and Misner and Sharp [46]).
The surface luminosity and the boundary red-shift z_Σ observed on Σ are

$$L_\Sigma = \frac{\kappa}{2}\{r^2U^3q\}_\Sigma \tag{7}$$

$$z_\Sigma = \left[1 + \frac{rU'}{U} + \frac{r\dot{U}}{X}\right]_\Sigma^{-1} - 1 \tag{8}$$

The total luminosity for an observer at rest at infinity is

$$L_\infty = -\frac{dM}{dv} = \frac{L_\Sigma}{(1+z_\Sigma)^2} \tag{9}$$

3. Solution of the field equations

In order to solve the Non-trivial Einstein's field equation which, we choose a particular form of the metric coefficients given in (1) into function of r and t coordinates as $X(r,t) = X_0(r)g(t)$ and $U(r,t) = U_0(r)f(t)$. The coupling constant in geometrized units is taken as $k = 8\pi(i.e. G = c = 1)$ and we get the following expressions for field equations.

$$\epsilon = \frac{\epsilon_0}{f^2} + \frac{3\dot{f}^2}{X^2{}_0g^2f^2} \tag{10}$$

$$(p_r)_0 = \frac{(p_r)_0}{f^2} + \frac{1}{X_0{}^2g^2}\left(-\frac{2\ddot{f}}{f} - \frac{\dot{f}^2}{f^2}\right) \tag{11}$$

$$(p_t)_0 = \frac{(p_t)_0}{f^2} + \frac{1}{X_0{}^2g^2}\left(-\frac{2\ddot{f}}{f} - \frac{\dot{f}^2}{f^2}\right) \tag{12}$$

$$q = -\frac{2{X_0}'\dot{f}}{{X_0}^2{U_0}^2 g f^3} \tag{13}$$

where

$$\epsilon_0 = -\frac{1}{{U_0}^2}\left(\frac{2{U_0}''}{U_0} - \frac{{U_0}'^2}{{U_0}^2} + \frac{4{U_0}'}{rU_0}\right) \tag{14}$$

$$(p_r)_0 = \frac{1}{{U_0}^2}\left(\frac{{U_0}'^2}{{U_0}^2} + \frac{2{U_0}'}{rU_0} + \frac{2{X_0}'{U_0}'}{X_0 U_0} + \frac{2{X_0}'}{rX_0}\right) \tag{15}$$

$$(p_t)_0 = \frac{1}{{U_0}^2}\left(\frac{{U_0}''}{U_0} - \frac{{U_0}'^2}{{U_0}^2} + \frac{{U_0}'}{rU_0} + \frac{{X_0}''}{X_0} + \frac{{X_0}'}{rX_0}\right) \tag{16}$$

Here the primes and dots stand for differentiation with respect to r and t respectively.

In the absence of dissipative force the equation (5), $(p_r)_\Sigma = (qU)_\Sigma$, reduces to the condition $(p_0)_\Sigma = 0$ and yields at $r = r_\Sigma = R_\Sigma$

$$\frac{2\ddot{f}}{f} + \frac{\dot{f}^2}{f^2} - \frac{2\dot{g}\dot{f}}{gf} = \frac{2\alpha g\dot{f}}{f^2} \tag{17}$$

where

$$\alpha = \left(\frac{X_0'}{U_0}\right)_\Sigma \tag{18}$$

To solve the equation (17), by assuming $g(t) = f(t)$ Tewari [26], obtain the following solution

$$\dot{f} = 2\alpha f + \gamma\sqrt{f} \tag{19}$$

$$t = \frac{1}{\alpha}\ln\left(1 + \frac{2\alpha}{\gamma}\sqrt{f}\right) \tag{20}$$

We observed that the function $f(t)$ decreases monotonically from the value $f(t) = 1$ at $t = -\infty$ to $f(t) = 0$ at $t = 0$.

Using the equations (11) and (12), Tewari and Charan [1] obtained the following parametric class of solution

$$X_0 = a_2(1 + a_1 r^2)(1 + b_1 r^2)^{\frac{n}{l+1}} \tag{21 }$$

$$U_0 = b_2(1 + b_1 r^2)^{\frac{1}{l+1}} \tag{22}$$

$$\boldsymbol{\delta}(r) = \frac{(2n-2)\quad 4a_1 b_1 r^2}{(l+1)\,(1+a_1 r^2)(1+b_1 r^2)} \tag{23}$$

where $n,\ l,\ b_1,\ b_2,\ a_1$ and a_2 are constants and

$$n = \frac{1}{2}\left\{(l+3) \pm (l^2 + 10l + 17)^{\frac{1}{2}}\right\} \tag{24}$$

where n is real if $l \geq -5 + 2\sqrt{2}$ or $l \leq -5 - 2\sqrt{2}$.

For different values of n or l Eqs. (21) and (22) give the variety of the solutions. Since many solutions can be obtained with the above related parametric class of solution, so keeping this point in mind we here find some exact solutions and listed them in a table and they will be fruitful for further study to construct the various models of radiating and static models.

Table1: Parametric class of solutions

S.N.	n	l	X_0	U_0	$\delta(r)$
1.	$-\frac{5}{2}$	$-\frac{47}{6}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{15}{41}}$	$(1+b_1r^2)^{-\frac{6}{41}}$	$\frac{168a_1b_1r^2}{41(1+a_1r^2)(1+b_1r^2)}$
2.	$-\frac{7}{3}$	$-\frac{47}{6}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{14}{41}}$	$(1+b_1r^2)^{-\frac{6}{41}}$	$\frac{216a_1b_1r^2}{41(1+a_1r^2)(1+b_1r^2)}$
3.	$-\frac{8}{7}$	$-\frac{124}{7}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{8}{117}}$	$(1+b_1r^2)^{-\frac{7}{117}}$	$\frac{120a_1b_1r^2}{117(1+a_1r^2)(1+b_1r^2)}$
4.	$-\frac{11}{3}$	$-\frac{101}{12}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{44}{89}}$	$(1+b_1r^2)^{-\frac{12}{89}}$	$\frac{448a_1b_1r^2}{89(1+a_1r^2)(1+b_1r^2)}$
5.	-13	$-\frac{103}{6}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{78}{97}}$	$(1+b_1r^2)^{-\frac{6}{97}}$	$\frac{672a_1b_1r^2}{97(1+a_1r^2)(1+b_1r^2)}$
6.	$1\pm\sqrt{2}$	-1	$a_2(1+a_1r^2)$	$(1+br^2)^0$	0
7.	$1\pm\sqrt{14}$	3	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{3\pm\sqrt{14}}{4}}$	$(1+b_1r^2)^4$	$\frac{2(2\pm\sqrt{14})a_1br^2}{(1+a_1r^2)(1+b_1r^2)}$
8.	$-\frac{7}{2}$	$-\frac{83}{10}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{35}{73}}$	$(1+b_1r^2)^{-\frac{10}{73}}$	$\frac{360a_1b_1r^2}{73(1+a_1r^2)(1+b_1r^2)}$
9.	$2\pm\sqrt{5}$	$\frac{(-1\pm\sqrt{5})}{2}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{3\pm\sqrt{5}}{2}}$	$(1+b_1r^2)^{-\frac{1\pm\sqrt{5}}{2}}$	$\frac{16a_1b_1r^2}{(1+a_1r^2)(1+b_1r^2)}$
10.	$-\frac{9}{5}$	$-\frac{83}{10}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{18}{73}}$	$(1+b_1r^2)^{-\frac{10}{73}}$	$\frac{224a_1b_1r^2}{73(1+a_1r^2)(1+b_1r^2)}$
11.	$-\frac{7}{6}$	$-\frac{103}{6}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{7}{97}}$	$(1+b_1r^2)^{-\frac{6}{97}}$	$\frac{104a_1b_1r^2}{97(1+a_1r^2)(1+b_1r^2)}$
12.	$-\frac{10}{9}$	$-\frac{208}{9}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{10}{199}}$	$(1+b_1r^2)^{-\frac{9}{199}}$	$\frac{152a_1b_1r^2}{199(1+a_1r^2)(1+b_1r^2)}$
13.	$-\frac{10}{7}$	$-\frac{83}{7}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{10}{76}}$	$(1+b_1r^2)^{-\frac{7}{76}}$	$\frac{136a_1b_1r^2}{76(1+a_1r^2)(1+b_1r^2)}$
14.	$1\pm\sqrt{5}$	$-7\pm3\sqrt{5}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{7\pm3\sqrt{5}}{3}}$	$(1+b_1r^2)^{\frac{2\pm\sqrt{5}}{3}}$	$\frac{8\sqrt{5}(\sqrt{5}-2)a_1b_1r^2}{3(1+a_1r^2)(1+b_1r^2)}$
15.	$-\frac{11}{5}$	$-\frac{118}{15}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{33}{103}}$	$(1+b_1r^2)^{-\frac{15}{103}}$	$\frac{384a_1b_1r^2}{103(1+a_1r^2)(1+b_1r^2)}$
16.	$-1\pm\sqrt{5}$	$-\frac{(7\sqrt{5}\pm25)}{2}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{-15\pm13\sqrt{5}}{31}}$	$(1+b_1r^2)^{-\frac{20\pm7\sqrt{5}}{31}}$	$\frac{8\sqrt{5}(\sqrt{5}\pm6)a_1b_1r^2}{31(1+a_1r^2)(1+b_1r^2)}$
17.	$-\frac{12}{11}$	$-\frac{178}{11}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{132}{167}}$	$(1+b_1r^2)^{-\frac{11}{167}}$	$\frac{1144a_1b_1r^2}{167(1+a_1r^2)(1+b_1r^2)}$
18.	$2\pm\sqrt{3}$	$\frac{(-3\pm2\sqrt{3})}{3}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{3\pm2\sqrt{3}}{2}}$	$(1+b_1r^2)^{\pm\frac{3}{2\sqrt{3}}}$	$\frac{4(3\pm\sqrt{3})a_1b_1r^2}{89(1+a_1r^2)(1+b_1r^2)}$
19.	$-\frac{1}{3}$	$-\frac{4}{3}$	$a_2(1+a_1r^2)(1+b_1r^2)^1$	$(1+b_1r^2)^{-3}$	$\frac{32a_1b_1r^2}{89(1+a_1r^2)(1+b_1r^2)}$
20.	$-\frac{13}{11}$	$-\frac{178}{11}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{13}{167}}$	$(1+b_1r^2)^{-\frac{11}{167}}$	$\frac{192a_1b_1r^2}{167(1+a_1r^2)(1+b_1r^2)}$
21.	$-\frac{7}{4}$	$-\frac{307}{12}$	$a_2(1+D_1r^2)(1+C_1r^2)^{\frac{28}{89}}$	$(1+b_1r^2)^{-\frac{12}{89}}$	$\frac{352a_1b_1r^2}{89(1+a_1r^2)(1+b_1r^2)}$
22.	$-\frac{17}{13}$	$-\frac{307}{26}$	$a_2(1+a_1r^2)(1+b_1r^2)^{\frac{34}{281}}$	$(1+b_1r^2)^{-\frac{12}{89}}$	$\frac{480a_1b_1r^2}{281(1+a_1r^2)(1+b_1r^2)}$

4. Physical analysis of the model

In order to construct the realistic model of collapsing radiating star, we assume $n = -\frac{4}{3}$, and from (21) and (22) we obtain

$$X_0 = a_2(1 + a_1r^2)(1 + b_1r^2)^{\frac{4}{31}} \tag{25}$$

$$U_0 = b_2(1 + b_1r^2)^{-\frac{3}{31}} \tag{26}$$

$$\delta(r) = \frac{56a_1b_1r^2}{31(1+a_1r^2)(1+b_1r^2)} \tag{27}$$

Using (14)-(16), (25) and (26) we have

$$\epsilon_0 = \frac{12b_1}{961{b_2}^2(1+b_1r^2)^{\frac{56}{31}}}[93 + 28b_1r^2] \tag{28}$$

$$(p_r)_0 = \frac{4b_1}{961{b_2}^2(1+b_1r^2)^{\frac{56}{31}}}\left[(31 + 16b_2r^2) + \frac{31a_1(1+b_1r^2)}{b_1(1+a_1r^2)}(31 + 25b_1r^2)\right] \tag{29}$$

$$(p_t)_0 = \frac{4b_1}{961{b_2}^2(1+br^2)^{\frac{56}{31}}}\left[(31 + 16b_2r^2) + \frac{31a_1(1+b_1r^2)}{b_1(1+a_1r^2)}(31 + 39b_1r^2)\right] \tag{30}$$

The junction condition$\{(p_r)_0\}_\Sigma = 0$ gives

$$a_1 = \frac{-b_1(31+16b_1{r_\Sigma}^2)}{31(1+b_1{r_\Sigma}^2)(31+25b_1{r_\Sigma}^2)+b_1{r_\Sigma}^2(31+16b_1{r_\Sigma}^2)} \tag{31}$$

Keeping in mind the conditions of physically reasonable solution at the centre we have the following suitable choice of constants $b_1 > 0$, $b_2 > 0$, and $\frac{-b_1}{31} < a_1 < 0$.

The total energy inside Σ for the static system is

$$m_0 = \frac{6b_1b_2{r_\Sigma}^3}{961{b_2}^2(1+b_1r^2)^{\frac{65}{31}}}[31 + 29b_1{r_\Sigma}^2] \tag{32}$$

The explicit expressions for ϵ, $p_{r,}$ $p_{t,}$ q and Θ reduces the following

$$\epsilon = \frac{\epsilon_0}{f^2} + \frac{12\beta^2(1-\sqrt{f})^2}{f^3\left[a_2(1+a_1r^2)(1+b_1r^2)^{\frac{4}{31}}\right]^2} \tag{33}$$

$$p_r = \frac{(p_r)_0}{f^2} + \frac{4\beta^2(1-\sqrt{f})}{f^{\frac{5}{2}}\left[a_2(1+a_1r^2)(1+b_1r^2)^{\frac{4}{31}}\right]^2} \tag{34}$$

$$p_t = \frac{(p_t)_0}{f^2} + \frac{4\beta^2(1-\sqrt{f})}{f^{\frac{5}{2}}\left[a_2(1+a_1r^2)(1+b_1r^2)^{\frac{4}{31}}\right]^2} \tag{35}$$

$$q = \frac{4r[4b_1+a_1(31+35C_1r^2)]}{31a_2{b_2}^2(1+a_1r^2)^2(1+b_1r^2)^{\frac{29}{31}}}\,\frac{2\beta(1-\sqrt{f})}{f^{\frac{7}{2}}} \tag{36}$$

The fluid collapse rate is given by

$$\Theta = \frac{-6\beta(1-\sqrt{f})}{f^{\frac{3}{2}}\left[a_2(1+a_1r^2)(1+b_1r^2)^{\frac{4}{31}}\right]} \tag{37}$$

where

$$\alpha = -\frac{2a_2r_\Sigma}{31b_2(1+b{r_\Sigma}^2)^{\frac{24}{31}}}[4b_1 + a_1(31 + b_1{r_\Sigma}^2)] \tag{38}$$

We can see the physical parameters $\epsilon, p_{r,} p_t$ are finite, positive, monotonically decreasing and their derivatives at any instant with respect to radial coordinate are negative for $0 \le r \le r_\Sigma$. Initially collapse is zero and it becomes infinite at the final phase of the configuration. The total energy entrapped inside Σ is given by

$$M(v) = \left[\frac{8}{961}\frac{b_2 r_\Sigma{}^5\left\{4b_1 + a_1\left(31+35b_1 r_\Sigma{}^2\right)^2\right\}}{(1+b_1 r_\Sigma{}^2)^{\frac{65}{31}}(1+a_1 r_\Sigma{}^2)^2}\left(1-\sqrt{f}\right)^2 + m_0 f\right] \qquad (39)$$

The luminosity and red shift observed on Σ and the luminosity observed by distant observer are given by

$$L_\Sigma = \frac{8}{961}\frac{r_\Sigma{}^4\left\{4b_1 + a_1\left(31+35b_1 r_\Sigma{}^2\right)\right\}^2}{(1+b_1 r_\Sigma{}^2)^2(1+a_1 r_\Sigma{}^2)^2}\frac{\left(1-\sqrt{f}\right)^2}{\sqrt{f}} \qquad (40)$$

$$L_\infty = \frac{8}{961}\frac{r_\Sigma{}^4\left\{4b_1 + a_1\left(31+35b_1 r_\Sigma{}^2\right)\right\}^2}{(1+b_1 r_\Sigma{}^2)^2(1+a_1 r_\Sigma{}^2)^2}\frac{\left(1-\sqrt{f}\right)^2}{\sqrt{f}}\frac{1}{(1+Z_\Sigma)^2} \qquad (41)$$

$$z_\Sigma = \left[\frac{(31+35b_1 r_\Sigma{}^2)}{31(1+br_\Sigma{}^2)} - \frac{4r_\Sigma{}^2\left\{4b_1 + a_1\left(31+35b_1 r_\Sigma{}^2\right)\right\}}{(1+b_1 r_\Sigma{}^2)(1+a_1 r_\Sigma{}^2)}\frac{\left(1-\sqrt{f}\right)}{\sqrt{f}}\right]^{-1}_\Sigma - 1 \qquad (42)$$

The expression (41) shows that L_∞ vanishes in the beginning when $f(t) \rightarrow 1$ and at the stage when $z_\Sigma \rightarrow \infty$.

We obtain the black hole formation time as

$$\sqrt{f_{BH}} = \frac{4r_\Sigma{}^2\left[4b_1 + a_1\left(31+35b_1 r_\Sigma{}^2\right)\right]}{4r_\Sigma{}^2[4b_1 + a_1(31+35b_1 r_\Sigma{}^2)] + (31+25b_1 r_\Sigma{}^2)(1+a_1 r_\Sigma{}^2)} \qquad (43)$$

$$t_{BH} = \frac{1}{\alpha} ln \left[\frac{(31+25b_1 r_\Sigma{}^2)(1+a_1 r_\Sigma{}^2)}{4r_\Sigma{}^2[4b_1 + a_1(31+35b_1 r_\Sigma{}^2)] + (31+25b_1 r_\Sigma{}^2)(1+a_1 r_\Sigma{}^2)}\right] \qquad (44)$$

The effective surface temperature observed by external observer can be calculated from the expression Tewari and Charan [1] as

$$T_\Sigma{}^4 = \frac{8}{961\pi\delta b_2{}^2}\frac{r_\Sigma{}^2\left(31+28b_1 r_\Sigma{}^2\right)^2}{(1+b_1 r_\Sigma{}^2)^{\frac{56}{31}}(1+a_1 r_\Sigma{}^2)^2}\frac{\left(1-\sqrt{f}\right)}{f^{\frac{5}{2}}}\frac{1}{(1+Z_\Sigma)^2} \qquad (45)$$

where for the photon δ is given by

$$\delta = \frac{\pi^2 k^4}{15h^3} \qquad (46)$$

where k and $\hbar$ denoting respectively Boltzmann and Plank constants.

The detailed description of temperature inside the star is given by Tewari and Charan [1], here the specific expressions for them are given as

$$T^4 = \left[\frac{T_0(t)}{\left[a_2(1+a_1 r_\Sigma{}^2)(1+b_1 r_\Sigma{}^2)^{\frac{4}{31}}\right]^4} - \frac{16\alpha}{3\varphi\left[a_2(1+a_1 r_\Sigma{}^2)(1+b_1 r_\Sigma{}^2)^{\frac{4}{31}}\right]}\frac{\left(1-\sqrt{f}\right)}{f^{\frac{3}{2}}}\right] \qquad (47)$$

where

$$T_0(t) = \left\{\frac{16\alpha}{3k\varphi}\frac{\left[a_2(1+a_1 r_\Sigma{}^2)(1+b_1 r_\Sigma{}^2)^{\frac{4}{31}}\right]^3\left(1-\sqrt{f}\right)}{f^{\frac{3}{2}}}\right\}_\Sigma + \left\{\frac{2\alpha\left[a_2(1+a_1 r_\Sigma{}^2)(1+br_\Sigma{}^2)^{\frac{4}{31}}\right]^2\left(1-\sqrt{f}\right)}{\pi\delta \mathrm{r}_\Sigma{}^2 f^{\frac{5}{2}}}\right\}_\Sigma \frac{1}{(1+Z_\Sigma)^2} \qquad (48)$$

It follows that the surface temperature of the collapsing star tends to zero at the beginning of the collapse $[f \rightarrow 1]$ and the stage of formation of black hole$[z_\Sigma \rightarrow \infty]$.

5. Concluding remarks

We have presented a new radiating star model for collapsing, spherically symmetric, shear-free, dissipative fluid distribution with a anisotropy pressure and radiating its energy in the form of radial heat flow corresponding to $n = -4/3$ of Tewari and Charan [1].The model obeyed all the relevant physical and thermodynamic conditions corresponding to non-negative expressions for density, tangential and radial pressures and radiation heat flux throughout the fluid sphere. The apparent luminosity as observed by the distant observer at rest at infinity is zero at the distant when collapse begins and at the stage when collapsing configuration reaches the horizon of black hole. Applications of our work is not limited only in theoretical study but also in observational work such as one can construct various models for compact objects by using these solutions.

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