SRMS Journal of Mathematical Sciences, Vol-4, 2018, pp. 9-19
ISSN: 2394-725X

An Improved Intuitionistic Fuzzy Choquet Integral Operator for Multi-criteria Decision-making Problems

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DOI: https://doi.org/10.29218/srmsjoms.

Keywords: Fuzzy measure, Intuitionistic fuzzy sets (IFSs), Choquet integral operator, Multi-criteria decision making (MCDM), Portfolio analysis.

Abstract

Aggregation operators based additive measures are not considered appropriate tools to aggregate the inter-dependent or interactive characteristics of criteria in multi-criteria decision-making problem. It would be more suitable to apply fuzzy measures to approximate human subjective decision-making process when the additivity and independence among the decision-making criteria are not necessary. In this paper, we propose a new intuitionistic fuzzy Choquet integral operator for multi-criteria decision-making problem with interaction phenomena among the decision making criteria. We introduce two operational laws on intuitionistic fuzzy values. Based on these operational laws and fuzzy measure, an intuitionistic fuzzy Choquet integral operator is proposed. Algorithm of multi-criteria decision making based on the proposed intuitionistic fuzzy Choquet integral operator is presented. A real-life example of ranking is also provided to illustrate the developed approach in multi-criteria decision making. To show the superiority of the proposed intuitionistic fuzzy Choquet integral operator, it is implemented in portfolio selection problem and portfolios are analyzed for their return and risk.

1. Introduction

Atanassov introduced intuitionistic fuzzy sets (IFSs), which emerged from the simultaneous consideration of membership and non-membership degrees with a degree of hesitation [1, 2]. IFSs are more suitable for dealing with fuzziness, uncertainty than the fuzzy sets developed by Zadeh [38]. The concept of vague sets was also introduced as the generalization of fuzzy sets but later on proved to be the same as that of IFS [5, 11].

IFSs along with intuitionistic fuzzy preference relations, were used to solve group decision-making problems and multi-criteria decision-making problems [26, 27]. Gabriella et al. [10] formed a generalized net model for multi-person multi-criteria decision-making process which was based on intuitionistic fuzzy graphs. An intuitionistic fuzzy interpretation of multi-person multi-criteria decision making is given by Atanassov et al. [4]. Li ([19]) investigated multi-attribute decision making using intuitionistic fuzzy sets and proposed several linear programming models to calculate optimal weights for criteria.

The concept of general IFS with triangular norm-based hesitation degrees was also introduced to build a comprehensive family of general algorithms of group decision making with a majority defined via linguistic quantifiers [23]. Fuzzy decision-making method based on weighted correlation coefficient under the intuitionistic fuzzy environment was also proposed [37]. Xidonas et al. [30] applied multi-criteria decision making to common stock portfolio selection problem using the prices of different stocks at the Athens Stock Exchange. Intuitionistic fuzzy point operators and a series of new score functions for the multi-attribute decision-making problems are also found in the literature [21]. Xu and Yager ([35]) developed some geometric aggregation operators (e.g., intuitionistic fuzzy weighted geometric (IFWG), intuitionistic fuzzy ordered weighted geometric (IFOWG), intuitionistic fuzzy hybrid geometric (IFHG)), and implemented the IFHG operator to multi-criteria decision-making problems with intuitionistic fuzzy information. A method for handling multi-criteria fuzzy decision-making problems based on intuitionistic fuzzy sets and linear programming model is also proposed by Lin et al. [20]. Various aggregation operators such as the intuitionistic fuzzy weighted averaging (IFWA), intuitionistic fuzzy ordered weighted averaging (IFOWA) and intuitionistic fuzzy hybrid aggregation (IFHA) and studied their various properties by [32-34].

Some other geometric and arithmetic interval-valued intuitionistic fuzzy aggregation operators were developed and used in MCDM with score and accuracy functions [34-36]. Recently some methods based on intuitionistic fuzzy weighted entropy and induced generalized intuitionistic fuzzy OWA operators are also found in the literature [25, 31].

Aggregation operators defined by [32] are based on the assumption that the criteria and preferences of decision makers are independent, which is characterized by an independence axiom [17, 29]. Therefore, these operators are based on the implicit assumption that the criteria and preference of decision makers are independent of one another; their effects are viewed as additive. But for real decision-making problems, there is always some degree of inter-dependent characteristics between attributes. Usually, there is an interaction among preference of decision makers. Hence this assumption is strongly taken into consideration to match decision behaviors of decision makers in the real world problems. In general, the independency among the criteria and preferences cannot be satisfied. To overcome this limitation, we define an intuitionistic fuzzy Choquet integral operator motivated by [7, 9]. Tan and Chen [28] introduced intuitionistic fuzzy Choquet integral operator for aggregating intuitionistic fuzzy information in multi-criteria decision making, and investigate its various properties.

In this paper, we propose a new intuitionistic fuzzy Choquet integral operator for aggregating intuitionistic fuzzy information in multi-criteria decision making. An approach to multi-criteria decision making with proposed intuitionistic fuzzy Choquet integral operator is also presented. A portfolio selection problem is also taken to show the superiority of the proposed intuitionistic fuzzy Choquet integral operator over [28].

2. Preliminaries

In this section, some basic definitions related to intuitionistic fuzzy sets are briefly introduced.

Definition 1. Let *U* be a fixed non-empty set, an intuitionistic fuzzy set *I* in *U* is defined as an object of the following form [1-3]:

$$I = \{\langle x, t_I(x), f_I(x)\rangle \quad \forall\, x \in U\} \tag{1}$$

where the functions $t_I : U \to [0,1]$ and $f_I : U \to [0,1]$ define the "degree of membership" and the "degree of non-membership" of the element $x \in U$ respectively with the following condition:

$$0 \le t_I(x) + f_I(x) \le 1, \quad \forall\, x \in U \tag{2}$$

In this paper, we call $\alpha = (t_\alpha, f_\alpha)$ an intuitionistic fuzzy value. Let Ω be the set of all intuitionistic fuzzy values on *U*. For every two intuitionistic fuzzy values *A* and *B* the following relations are valid:

1. $A = B$ if and only if $t_A(x) = t_B(x)$ and $f_A(x) = f_B(x)$ for all $x \in U$. (3)
2. $A \le B$ if and only if $t_A(x) \le t_B(x)$ and $f_A(x) \ge f_B(x)$ for all $x \in U$. (4)

However, the second condition is not satisfied in most of the situations. So it cannot be used to compare intuitionistic fuzzy numbers. Score function [6] and accuracy function [14] are used to compare for the comparison of two intuitionistic fuzzy numbers.

Definition 2. Let $a = (t_a, f_a)$ and $b = (t_b, f_b)$ be two intuitionistic fuzzy numbers, then $S(a) = t_a - f_a$, $S(b) = t_b - f_b$ are the score functions and $H(a) = t_a + f_a$ $H(b) = t_b + f_b$, are the accuracy functions of *a* and *b*.

- If $S(a) < S(b)$ then *a* is smaller than *b*, denoted by $a < b$.
- If $S(a) = S(b)$, then

(1) If $H(a) < H(b)$ then *a* is smaller than *b*, denoted by $a < b$.

(2) If $H(a) = H(b)$ then *a* and *b* represent the same information, denoted by $a = b$.

Motivated by the various operators defined by many researchers, we define two following operational laws for intuitionistic fuzzy numbers [3, 8, 35]:

Definition 3. Let $a=(t_a, f_a)$ and $b=(t_b, f_b)$ be two intuitionistic fuzzy numbers, then

(i) $a \oplus b = (t_a t_b, f_a f_b - f_a f_b)$ (5)

(ii) $\lambda a = \left((t_a)^\lambda, 1-(1-f_a)^\lambda\right)$, where $\lambda > 0$ (6)

Proposition 1. Let $a=(t_a, f_a)$ and $b=(t_b, f_b)$ be two intuitionistic fuzzy values and let $c = a \oplus b$ and $d = \lambda a, \text{where } \lambda > 0$; then both *c* and *d* are also intuitionistic fuzzy values.

Proof: Since $a=(t_a, f_a)$ and $b=(t_b, f_b)$ are two intuitionistic fuzzy values, we have

$t_a \in [0,1], f_a \in [0,1], t_b \in [0,1], f_b \in [0,1], and\, t_a + f_a \le 1, t_b + f_b \le 1.$ Then

$$t_a.t_b \ge 0,\ f_a + f_b - f_a.f_b \ge 0,$$
$$t_a.t_b + f_a + f_b - f_a.f_b \le t_a.t_b + (1-t_a) + (1-t_b) - (1-t_a).(1-t_b) = 1,$$

$$and\ (t_a)^\lambda \ge 0,\quad 1-(1-f_a)^\lambda \ge 0,\quad \text{where } \lambda > 0$$
$$(t_a)^\lambda + 1 - (1-f_a)^\lambda \le (1-f_a)^\lambda + 1 - (1-f_a)^\lambda = 1$$

Thus it is clear that two operational laws defined above are also intuitionistic fuzzy numbers.

Proposition 2. Let $a=(t_a, f_a)$ and $b=(t_b, f_b)$ be two intuitionistic fuzzy numbers and for every $\lambda_1, \lambda_2 > 0$. Then we have

(i) $a \oplus b = b \oplus a,$

(ii) $\lambda_1(a \oplus b) = \lambda_1 a \oplus \lambda_1 b,$

(iii) $\lambda_1 a \oplus \lambda_2 a = (\lambda_1 + \lambda_2)a.$

Definition 4. Let $A=(a_{ij})_{m\times n}$, where $a_{ij} = (t_{ij}, f_{ij}), (i=1,2,\dots\dots, m, j=1,2,3\dots\dots, n)$ to be the intuitionistic fuzzy values decision making and its equivalent intuitionistic fuzzy preference relation is $B=(b_{ij})_{m\times n}\left(here, b_{ij} = (t_{ij}, 1-f_{ij})\right)$, if there exists a vector $w=(w_1, w_2, \dots, w_n)^T$, for all values of *i*, *j* such that

$$t_{ij} \le 0.5(w_i - w_j + 1) \le 1 - f_{ij} \text{ for all } i, j \tag{7}$$

with the conditions $\sum_{i=1}^{n} w_i = 1$ and $w_i \ge 0$ (8)

Then *B* is a consistent intuitionistic fuzzy preference relation, otherwise an inconsistent intuitionistic fuzzy preference relation [33].

3. Intuitionistic Fuzzy Choquet Integral Operator

3.1. Fuzzy Measure and Choquet Integral

The fuzzy measure given by Sugeno follows monotonic property instead of additive property. In decision-making problems, it does not need to assume that criteria or preferences are independent of each other, and it was used as a powerful tool for modeling interaction phenomena among criteria in decision making. Many researchers successfully resolved interaction phenomena among criteria [12, 13, 15, 18]. As an aggregation operator, the Choquet integral has been proposed by many authors as an adequate substitute to the weighted arithmetic mean or OWA operator to aggregate interacting criteria. In OWA model, each criterion is given a weight between zero and one, which represents the importance of the criterion in the decision making. In the Choquet integral model, a fuzzy measure is used to define a weight on each combination of criteria, where criteria can be dependent. Thus it is possible to make a model in which interaction among criteria is considered.

Definition 5. A fuzzy measure on *X* is a function $\mu : P(X) \rightarrow [0,1]$, satisfying the following properties:

(i) $\mu(\phi) = 0,\ \mu(X) = 1$ (Boundary conditions)

(ii) If $A, B \in P(X)$ and $A \subseteq B$ then $\mu(A) \leq \mu(B)$ (monotonicity)

Definition 6 Let *F* be a positive real-valued function on *X*, and μ be a fuzzy measure on *X*. The discrete Choquet integral of *f* with respective to μ is defined as follows [13]:

$$C_\mu(F) = \sum_{i=1}^{n} F_{(i)} \left[\mu\left(A_{(i)}\right) - \mu\left(A_{(i+1)}\right)\right] \tag{9}$$

where (.)indicates a permutation on X such that $F_{(1)} \leq F_{(2)} \leq F_{(3)} \leq .... \leq F_{(n)}$. Also

$$A_{(i)} = (i, i+1, ..., n), A_{(n+1)} = \phi \tag{10}$$

3.2. Intuitionistic fuzzy Choquet Integral Operator

Intuitionistic fuzzy Choquet integral operator is defined as follows [28]:

Definition 7. Let *X* be the collection of *n* intuitionistic fuzzy numbers on *X*, defined as $a_i = (t_{a_i}, f_{a_i})$ (*i* = 1, 2, …., *n*), and μ be a fuzzy measure on *X*. Then the (discrete) intuitionistic fuzzy Choquet integral of intuitionistic fuzzy number a_i with respective to μ is given by the following expression:

$$\text{IFC}_\mu(a_1, a_2, ..., a_n) = \sum_{i=1}^{n} a_{(i)} \left(\mu(A_{(i)}) - \mu(A_{(i+1)})\right) \tag{11}$$

where (.) indicates a permutation on *X* such that $a_{(1)} \leq a_{(2)} \leq a_{(3)} \leq .... \leq a_{(n)}$ Also

$$A_{(i)} = (i, i+1, ..., n), A_{(n+1)} = \phi.$$

Theorem 1: Let $a_i = (t_{a_i}, f_{a_i})$ (*i* =1, 2, …., *n*) be a collection of n intuitionistic fuzzy numbers on *X*, and μ be a fuzzy measure on X. Then their aggregated value by using the IFC_μ operator is also an intuitionistic fuzzy number, and

$$IFC_{\mu}(a_1, a_2, ..., a_n) = \left(\prod_{i=1}^{n} (t_{a_{(i)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})}, 1 - \prod_{i=1}^{n} (1 - f_{a_{(i)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})} \right).$$

where (.) indicates a permutation on X such that $a_{(1)} \le a_{(2)} \le a_{(3)} \le .... \le a_{(n)}$ Also

$$A_{(i)} = (i, i+1, ..., n), A_{(n+1)} = \phi.$$

Proof: Here we prove this theorem by using mathematical induction on the collection of intuitionistic fuzzy numbers on *X* i.e., *n*.

For *n* = 2, from the operational laws of definition 3, we have

$$\left(\mu(A_{(1)}) - \mu(A_{(2)})\right) a_{(1)} = \left((t_{a_{(1)}})^{\mu(A_{(1)})-\mu(A_{(2)})}, 1 - (1 - f_{a_{(1)}})^{\mu(A_{(1)})-\mu(A_{(2)})} \right).$$

$$\left(\mu(A_{(2)}) - \mu(A_{(3)})\right) a_{(2)} = \left((t_{a_{(2)}})^{\mu(A_{(2)})-\mu(A_{(3)})}, 1 - (1 - f_{a_{(2)}})^{\mu(A_{(2)})-\mu(A_{(3)})} \right).$$

We have

$$a \oplus b = \left(t_a.t_b, f_a + f_b - f_a f_b\right) = \left(t_a.t_b, 1-(1-f_a).(1-f_b)\right)$$

So,

$$IFC_{\mu}(a_1, a_2,) = a_{(1)}\left(\mu(A_{(1)}) - \mu(A_{(2)})\right) \oplus a_{(2)}\left(\mu(A_{(2)}) - \mu(A_{(3)})\right)$$
$$= \left((t_{a_{(1)}})^{\left(\mu(A_{(1)})-\mu(A_{(2)})\right)}.(t_{a_{(2)}})^{\left(\mu(A_{(12)}-\mu(A_{(3)})\right)}, \left(1-(1-f_{a_{(1)}})^{\mu(A_{(1)})-\mu(A_{(2)})}\right).\left(1-(1-f_{a_{(2)}})^{\mu(A_{(2)})-\mu(A_{(3)})}\right) \right)$$

It is clear that for *n* = 2, the result holds.

Let for *n* = *k*, the result be true i.e.

$$IFC_{\mu}(a_1, a_2, ..., a_k) = \left(\prod_{i=1}^{k} (t_{a_{(i)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})}, 1 - \prod_{i=1}^{k} (1 - f_{a_{(i)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})} \right).$$

For *n* = *k* +1, we have

$$IFC_{\mu}(a_1, a_2, ..., a_{k+1}) = \Big(\prod_{i=1}^{k} (t_{a_{(i)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})}.(t_{a_{(k+1)}})^{\mu(A_{(k+1)})-\mu(A_{(k+2)})},$$
$$1 - \prod_{i=1}^{k} (1 - f_{a_{(i)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})}.(1 - f_{a_{(k+1)}})^{\mu(A_{(k+1)})-\mu(A_{(k+2)})}\Big)$$
$$= \left(\prod_{i=1}^{k+1} (t_{a_{(i)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})}, 1 - \prod_{i=1}^{k+1} (1 - f_{a_{(i)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})} \right)$$

It confirms that for *n* = *k* +1, the result still holds, which completes the proof of theorem 1.

4. Method to Multi-criteria Decision Making with Proposed Intuitionistic Fuzzy Choquet Integral Operator

The process of finding the best alternative from all of the feasible alternatives where all the alternatives can be evaluated according to a number of criteria or attribute is called multi-criteria decision making (MCDM) problem. MCDM is also very useful to rank the available alternatives associated with multiple criteria that include includes uncertain and imprecise data and the information regarding alternatives. We first describe the multi-criteria decision-making problems under intuitionistic fuzzy environment.

For a multi-criteria decision-making problem, let $A=(a_1,a_2,a_3,.....,a_m)$ be a set of *m* alternatives and $C=(c_1,c_2,c_3,.....,c_n)$ be a set of *n* criteria. With the help of the performance of alternative a_i with respect to criteria c_j, fuzzy sets are formed and then by using the construction theorem given by [16] intuitionistic fuzzy sets corresponding to fuzzy sets are constructed. Performance of alternative a_i with respect to criteria c_j is measured by intuitionistic fuzzy numbers $a_{ij}=(t_{ij},f_{ij})(i=1,2,3,...,m;\ j=1,2,3,....,n),$ where t_{ij} indicates the degree that the alternative a_i satisfies the criteria c_j; f_{ij} indicates the degree that the alternative a_i does not satisfy the criteria c_j, and $t_{ij}\in[0,1],\ f_{ij}\in[0,1],\ 0\le t_{ij}+f_{ij}\le 1.$

The proposed multi-criteria decision-making method to get the ranking of alternatives is based on intuitionistic fuzzy Choquet integral operator defined in Theorem 1. From intuitionistic fuzzy score matrix, the optimal priority weight vector is calculated by using model proposed by [33].

5. Algorithm for MCDM Method with Proposed Intuitionistic Fuzzy Choquet Integral Operator

Algorithm of proposed intuitionistic fuzzy Choquet integral operator based MCDM method is described in the following steps:

Step 1. The partial evaluation of the alternative $a_i\ (i=1,2,3,.....,m)$ is made by an intuitionistic fuzzy value $a_{ij}=(t_{ij},f_{ij})(i=1,2,3,...,m;\ j=1,2,3,....,n),$ which are formed by using the construction method of intuitionistic fuzzy sets given by [16], with respect to criteria c_j (j = 1,2,3,.....,n). Then we can obtain an intuitionistic fuzzy values decision-making matrix as follow:

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & ..... & a_{1n} \\ a_{21} & a_{22} & a_{23} & ..... & a_{2n} \\ a_{31} & a_{32} & a_{33} & ..... & a_{3n} \\ ..... & ..... & ..... & ..... & ..... \\ a_{m1} & a_{m2} & a_{m3} & ..... & a_{mn} \end{bmatrix}=\begin{bmatrix} (t_{11},f_{11}) & (t_{12},f_{12}) & (t_{13},f_{13}) & ..... & (t_{1n},f_{1n}) \\ (t_{21},f_{21}) & (t_{22},f_{22}) & (t_{23},f_{23}) & ..... & (t_{2n},f_{2n}) \\ (t_{31},f_{31}) & (t_{32},f_{32}) & (t_{33},f_{33}) & ..... & (t_{3n},f_{3n}) \\ ..... & ..... & ..... & ..... & ..... \\ (t_{m1},f_{m1}) & (t_{m2},f_{m2}) & (t_{m3},f_{m3}) & ..... & (t_{mn},f_{mn}) \end{bmatrix}.$$

Step 2. Transform the intuitionistic fuzzy values decision-making matrix *A* obtained in Step 1 into its equivalent intuitionistic fuzzy preference relation $B=(b_{ij})_{m\times n}\ (here, b_{ij}=(t_{ij},1-f_{ij})),$ then solve the model proposed by [33], to obtain the weight w_i of each criteria c_i. If the weight w_i corresponding to any criteria is zero, discard the corresponding criteria from the intuitionistic fuzzy preference relation.

Step 3. Intuitionistic fuzzy values decision making matrix *A* is transformed into *A*' by discarding the criteria having weights zero. With the help of score functions, we calculate the score values $S(a_{ij})$ of the intuitionistic fuzzy number

a_{ij} of the alternative $a_i\ (i=1,2,3,....., m)$ If there is no difference between two score functions $S(a_{ij})$ and $S(a_{ik})$ then by accuracy function, we calculate $H(a_{ij})$ and $H(a_{ik})$ partial evaluation a_{ij} a_{ik} and are ranked by using score and accuracy function defined in definition 2.

Step 4. Weights w_i obtained in step 2 can be taken as the fuzzy measure of corresponding criteria as it satisfies all the postulates of fuzzy measure.

Step 5. Using the following intuitionistic fuzzy Choquet integral operator

$$a_i = IFC_{\mu}(a_{i1}, a_{i2}, ..., a_{in}) = \left(\prod_{j=1}^{n} (t_{a_{i(j)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})}, 1-\prod_{j=1}^{n} (1-f_{a_{i(j)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})} \right)$$

We evaluate the overall value $a_i\ (i=1,2,3,....., m)$ which is aggregate in the i^{th} row of the intuitionistic fuzzy values decision-making matrix, of the alternative $a_i\ (i=1,2,3,....., m)$.

Step 6. The overall values $a_i = (t_{a_i}, f_{a_i})$ obtained in Step 5 of the alternatives $a_i\ (i=1,2,3,....., m)$ are used to calculate the score functions or the accuracy functions to rank the alternatives $a_i\ (i=1,2,3,....., m)$ and select the best one.

6. A Numerical Example

In this section, we have taken four companies Steel Authority of India (a_1), Bajaj Steel (a_2) Infotech Enterprise (a_3) and Industrial Bank of India (a_4) for ranking on the basis of following five criteria $(c_1, c_2, c_3, c_4, c_5)$:

- **(i)** Earnings per share (EPS) of the company (c_1);
- **(ii)** Price per book (Price/Book) of the company (c_2);
- **(iii)** A dividend of the company (c_3);
- **(iv)** P/C ratio (Put-Call Ratio) of the company (c_4);
- **(v)** P/E ratio (price-to-earnings ratio) of the company (c_5);

The actual numerical values of each criteria of each company are retrieved from *http://www.moneycontrol.com* and are placed in the following table (Table 1).

Table 1

Actual numerical value of criteria

Criteria Company	c_1	c_2	c_3	c_4	c_5
a_1	9.61	1.07	24	7.26	9.97
a_2	9.85	0.52	20	3.26	11.33
a_3	8.80	1.70	25	11.42	15.79
a_4	18.50	0.79	35	5.11	5.47

Step 1. For each criterion, fuzzy sets are constructed and then intuitionistic fuzzy sets are constructed by using the construction theorem. Using constructed intuitionistic fuzzy sets, the following intuitionistic fuzzy values decision-making matrix is obtained:

$$A=(a_{ij})_{4\times 5}=\begin{bmatrix}(0.213,0.632) & (0.441,0.384) & (0.395,0.428) & (0.434,0.355) & (0.307,0.516)\\ (0.224,0.621) & (0.381,0.444) & (0.189,0.634) & (0.611,0.128) & (0.188,0.635)\\ (0.175,0.670) & (0.185,0.640) & (0.632,0.191) & (0.197,0.592) & (0.336,0.487)\\ (0.630,0.215) & (0.639,0.186) & (0.290,0.533) & (0.500,0.289) & (0.634,0.189)\end{bmatrix}.$$

Step 2. Transform the above intuitionistic fuzzy values decision-making matrix *A* into its equivalent intuitionistic fuzzy preference relation *B* and presented as:

$$B=(b_{ij})_{4\times 5}=\begin{bmatrix}(0.213,0.368) & (0.441,0.616) & (0.395,0.572) & (0.434,0.645) & (0.307,0.484)\\ (0.224,0.379) & (0.381,0.556) & (0.189,0.366) & (0.661,0.872) & (0.188,0.365)\\ (0.175,0.330) & (0.185,0.360) & (0.632,0.809) & (0.197,0.408) & (0.336,0.513)\\ (0.630,0.785) & (0.639,0.814) & (0.290,0.467) & (0.500,0.711) & (0.634,0.811)\end{bmatrix}.$$

The optimal priority weight vector is calculated by using the following linear programming model given by [33]:

$$Min\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\left(d_{ij}^{-}+d_{ij}^{+}\right)$$

subject to the following conditions:

$$0.5(w_i-w_j+1)+d_{ij}^{-}\geq \mu_{ij}$$
$$0.5(w_i-w_j+1)-d_{ij}^{+}\leq 1-\upsilon_{ij}$$
$$w_i\geq 0,\sum_{i=1}^{n}w_i=1, d_{ij}^{+}\geq 0, d_{ij}^{-}\geq 0$$

with the conditions $\sum_{i=1}^{4}w_i=1\ w_i\geq 0,\ d_{ij}^{+}\geq 0,\ d_{ij}^{-}\geq 0$,. Here, $d_{ij}^{\pm}$ are deviation values.

We use scilab to solve above LPP and to obtain optimal priority vector as $w=(0.222,0,0.170,0.354,0.254)^T$. Criterion c_2 is left as its weight is zero.

Step 3. Discarding the criterion c_2, the intuitionistic fuzzy values decision-making matrix *A* is reduced into *A'* and is given as follow:

$$A'=\begin{bmatrix}(0.213,0.632) & (0.395,0.428) & (0.434,0.355) & (0.307,0.516)\\ (0.224,0.621) & (0.189,0.634) & (0.661,0.128) & (0.188,0.635)\\ (0.175,0.670) & (0.632,0.191) & (0.197,0.592) & (0.336,0.487)\\ (0.630,0.215) & (0.290,0.533) & (0.500,0.289) & (0.634,0.189)\end{bmatrix}.$$

Score functions of the intuitionistic fuzzy numbers obtained in step 3 are calculated and rank these intuitionistic fuzzy numbers by using score functions or accuracy functions.

$$a_{1(1)} = (0.213, 0.632), a_{1(2)} = (0.307, 0.516), a_{1(3)} = (0.395, 0.428), a_{1(4)} = (0.434, 0.355),$$
$$a_{2(1)} = (0.188, 0.635), a_{2(2)} = (0.189, 0.634), a_{2(3)} = (0.224, 0.621), a_{2(4)} = (0.661, 0.128),$$
$$a_{3(1)} = (0.175, 0.670), a_{3(2)} = (0.197, 0.592), a_{3(3)} = (0.336, 0.487), a_{3(4)} = (0.632, 0.191),$$
$$a_{4(1)} = (0.290, 0.533), a_{4(2)} = (0.500, 0.289), a_{4(3)} = (0.630, 0.215), a_{4(4)} = (0.634, 0.189).$$

Step 4. The fuzzy measure of all possible combinations of criteria are as follows:

$\mu(c_1) = 0.222$	$\mu(c_3) = 0.170$	$\mu(c_4) = 0.354$	$\mu(c_5) = 0.254$
$\mu(c_1, c_3) = 0.392$	$\mu(c_1, c_4) = 0.576$	$\mu(c_1, c_5) = 0.476$	$\mu(c_3, c_4) = 0.524$
$\mu(c_3, c_5) = 0.424$	$\mu(c_4, c_5) = 0.608$	$\mu(c_1, c_3, c_4) = 0.746$	$\mu(c_1, c_3, c_5) = 0.646$
$\mu(c_1, c_4, c_5) = 0.830$	$\mu(c_3, c_4, c_5) = 0.778$	$\mu(c_1, c_3, c_4, c_5) = 1.0$	

Step 5. Use the following intuitionistic fuzzy Choquet integral operator to evaluate the overall value $a_i \ (i = 1,2,3,4)$.

$$a_i = IFC_\mu(a_{i1}, a_{i2}, \ldots\ldots, a_{in}) = \left(\prod_{j=1}^{n} (t_{a_{i(j)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})}, 1 - \prod_{j=1}^{n} (1 - f_{a_{i(j)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})} \right)$$

$$a_1 = IFC_\mu(a_{11}, a_{12}, a_{13}, a_{14}) = \left(\prod_{j=1}^{4} (t_{a_{1(j)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})}, 1 - \prod_{j=1}^{4} (1 - f_{a_{1(j)}})^{\mu(A_{(i)})-\mu(A_{(i+1)})} \right)$$

$$a_1 = (0.3379, 0.4802).$$

Similarly $a_2 = (0.2755, 0.5383), a_3 = (0.3116, 0.4977)\ and\ a_4 = (0.5107, 0.3064).$

Step 6. Calculate the score functions or the accuracy functions to rank the alternatives $a_i \ (i = 1,2,3,4)$.

$$S(a_1) = -0.1423, S(a_2) = -0.2628, S(a_3) = -0.1861\ and\ S(a_4) = 0.2043,$$

Thus, four companies are ranked as $a_4 > a_1 > a_3 > a_2$.

7. Conclusion

In this paper, we define two operational laws of intuitionistic fuzzy values and develop an intuitionistic fuzzy Choquet integral operator for multiple criteria decision making. Based on proposed intuitionistic fuzzy Choquet integral operator and model proposed by [33], we have presented a new method of multi-criteria decision-making problems under intuitionistic fuzzy environment, where inter-dependency among the decision making criteria is considered. An algorithm for multi-criteria decision making to rank the alternatives is proposed. An example is also taken to implement the proposed multi-criteria decision-making process to rank the alternatives.

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Received Date: 15 December 2018
Revised Date: 11 May 2019
Publication Date: 07 November 2019