

Reliability Estimation of Bridge System Using C-H-A Algorithm

Suneet Saxena

Assistant Professor, Department of Mathematics, SRMS College of Engineering & Technology, Bhojipura, Bareilly (U.P), India

Email: Sunarn2012@gmail.com

DOI : <http://dx.doi.org/10.29218/srmsmaths.v3i01.44>

ABSTRACT

Keywords

Reliability, C-H-A algorithm
, Structure function, Design
Matrix.

The paper present study of reliability of system consisting of electronic or mechanical components. These components perform different function and failure of any component causes failure of entire system. Every system has its life and failure is inevitable, therefore objective of reliability study is to ensure that system works up to its life without failure. In this paper we have applied C-H-A algorithm to bridge system and developed Reliability expression for it.

INTRODUCTION

Reliability is the science that studies the laws of occurrence of failure in technical equipments and offers remedial measures. Thus the basic concept of reliability is that failure free operation, which is defined as the ability of a device/system/equipment to function properly without a failure throughout a specified time and under specified conditions.

The study of reliability gained momentum after World War II. Following the war, Aeronautical Radio, Inc. (ARINC) was established by the Commercial Airlines to improve airborne electronic equipment. In 1950, the U.S. Air Force formed an ad hoc group to improve general equipment reliability and in 1952, the defence department established the Advisory Group on Reliability of Electronic Equipment (AGREE). The requirement of reliability testing and demonstration of new system emerged from this advisory group. During 1950s and 1960s, reliability studied by Bazovsky[4], Barlow and Proshan[3], Smith[7] and Kapoor and Lamberson[6]. Their work centered around the use of the various distribution such as exponential, Weibull, Normal etc. to represent failure times. During 1970s the forces shifted to fault tree analysis, largely because of concern about nuclear reactor safety.

In 1980s reliability networks received considerable attention in the literature. Both reliability and maintainability received renewed emphasis in the mid 1980s with the introduction of the Air Force Reliability and Maintainability (R&M) 2000 program. Objectives of the R&M 2000 program were to increase system readiness and availability and to reduce maintenance personnel requirements and life cycle cost through increased reliability and maintainability by the year 2000.

A very general alternative approach for analyzing the reliability of complex system is through the use of system structure function. Once one obtains the expression for the structure function, the system reliability computation becomes straight-forward. Such attempts have been made in the classical 1975 book by Barlow and Proshan[2]. Various algorithm have been developed to evaluate structure function. Aven algorithm [1] based on minimal cut sets. It depends on the initial choices of 2 parameters. C-H-A algorithm proposed by Chaudhari, Hu and Afsar[5] based on minimal path set .

In this paper we have applied C-H-A algorithm to bridge system and developed Reliability expression for it. The process involves construction of structure function and design matrix. The paper consist of six section. Section 1 is the introduction. Section 2 presents Notations and Definitions. Section 3 discussed CHA algorithm. Section 4 focuses on development of reliability of Bridge system. Finally sections 5 and 6 present discussion and references respectively.

Notations and Definitions

Notations

n : Number of components.

x_i : State of i^{th} component.

x : $((x_1, x_2, \dots, x_i, \dots, x_n))$ state of system consisting of n components.

$\emptyset(x)$: Structure function.

p_i : reliability of i^{th} component which is the probability.

R : Reliability of system.

Definitions

Structure Function:

$$\text{Let } x_i = \begin{cases} 1 & \text{if } i^{th} \text{ component operates} \\ 0 & \text{if } i^{th} \text{ component fails} \end{cases}$$

Then the system structure function is defined as

$$\emptyset(x) = \begin{cases} 1 & \text{if system operates} \\ 0 & \text{if system fails} \end{cases}$$

Reliability of System:

Reliability of system in terms of structure function is defined as

$$R = \text{probability} \{ \emptyset(x) = 1 \} = E\{ \emptyset(x) \}$$

Where Expectation E given by

$$E\{ \emptyset(x) \} = 0 \cdot \text{probability} \{ \emptyset(x) = 0 \} + 1 \cdot \text{probability} \{ \emptyset(x) = 1 \}$$

C-H-A Algorithm

Step 1 : Find out minimal path sets.

Step 2: Construct matrix P using minimal path set. Each column of P represent minimal path. Assign 1 for the component present in path set and 0 for the component not present in path set.

Step 3: Construct the design matrix D using the columns of P matrix. Start with two columns of P matrix and apply OR operation on a respective rows and resultant column will be appended in P matrix. Perform same operation on all remaining columns. Process will be extended to three columns and so on. Once process will stop we will obtain design matrix D .

Step 4: Construct a row vector S whose number of columns are same that of design matrix D . First m elements are 1's, where m is the number of columns in P matrix. Next elements (1 or -1) are determined according to rule $(-1)^{i-1}$, where i is the number of columns of P that are taken at a time to be OR'ed in particular step.

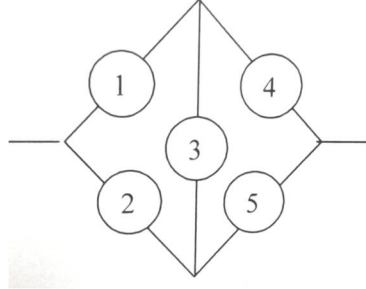
Step 5: Let m be the number of columns in P matrix. Construct the structure function as follows

$$\emptyset(x) = \sum_{j=1}^{2^m-1} S(j) \cdot \prod_{i=1}^n x_i^{D(i,j)}$$

Where $D(i,j)$ = element (i,j) of D , $S(j)$ = element j of S .

Reliability of Bridge System

The bridge structure has 5 components



Minimal path sets are : {1,4}, {2,5}, {1,3,5}, {2,3,4}.

P matrix is given by

$$P = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Design matrix is given by $D = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$

$$S = [1 \quad 1 \quad 1 \quad 1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad -1 \quad 1 \quad 1 \quad 1 \quad 1 \quad -1]$$

structure function $\emptyset(x) = \sum_{j=1}^{2^m-1} S(j) \cdot \prod_{i=1}^n x_i^{D(i,j)}$ $m=4, n=5$.

$$\begin{aligned} \emptyset(x) &= \sum_{j=1}^{15-1} S(j) \cdot \prod_{i=1}^5 x_i^{D(i,j)} \\ &= S(1) x_1^{D(1,1)} x_2^{D(2,1)} x_3^{D(3,1)} x_4^{D(4,1)} x_5^{D(5,1)} + \dots \\ &\dots + S(15) x_1^{D(1,15)} x_2^{D(2,15)} x_3^{D(3,15)} x_4^{D(4,15)} x_5^{D(5,15)}. \end{aligned}$$

$$\begin{aligned} \emptyset(x) &= x_1 x_4 + x_2 x_5 + x_1 x_3 x_5 + x_2 x_3 x_4 - x_1 x_2 x_4 x_5 - x_1 x_3 x_4 x_5 - x_1 x_2 x_3 x_4 - x_1 x_2 x_3 x_5 - x_2 x_3 x_4 x_5 \\ &\quad - 2x_1 x_2 x_3 x_4 x_5 \end{aligned}$$

Expectation

$$\begin{aligned} E\{\emptyset(x)\} &= E(x_1 x_4) + E(x_2 x_5) + E(x_1 x_3 x_5) + E(x_2 x_3 x_4) - E(x_1 x_2 x_4 x_5) - E(x_1 x_3 x_4 x_5) - E(x_1 x_2 x_3 x_4) \\ &\quad - E(x_1 x_2 x_3 x_5) - E(x_2 x_3 x_4 x_5) - 2E(x_1 x_2 x_3 x_4 x_5) \end{aligned}$$

Reliability

$$\begin{aligned} R &= p_1 p_4 + p_2 p_5 + p_1 p_3 p_5 + p_2 p_3 p_4 - p_1 p_2 p_4 p_5 - p_1 p_3 p_4 p_5 - p_1 p_2 p_3 p_4 - p_1 p_2 p_3 p_5 - p_2 p_3 p_4 p_5 - \\ &\quad 2p_1 p_2 p_3 p_4 p_5 \end{aligned}$$

DISCUSSION

For last four decades, various methods have been developed to evaluate reliability of the system. The method discussed in this paper has got its own importance. Being an algorithm approach, we can write computer program and use computer to evaluate reliability of complex system. Further, the important reliability measures such as Brinbaum Reliability and Chaudhari bounds can be evaluated easily.

REFERENCES

- [1] Aven, T. (1986) : Reliability/availability of Coherent system based on minimal cut sets, Reliability Engg., Vol. 13, pp. 93-104.
- [2] Barlow, R. E. and Proshan, F. (1975) : Statistical Theory of Reliability and Testing, Holt, Winston Richart.
- [3] Barlow, R. E. and Proshan, F. and Hunter, L. C. (1967): Mathematical Theory of Reliability, John Wiley & Sons, New York.
- [4] Bazovsky, I. (1961): Reliability Theory and Practice, Prentice Hall, New Jersey.
- [5] Chaudhari, Gopal, Hu, K. and Nadar, A.(2001) : A New Approach to System Reliability, IEEE Trans. On Reliability, Vol. 50, No. 1, pp 75-84.
- [6] Kapur, K. C. and Lamberson, L. R. (1977): Reliability in Engineering Design, John Wiley & Sons, New York.
- [7] Smith, C. O. (1976) : Introduction to Reliability Design, McGraw-Hill, New York.