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Exact Relativistic Modeling of Einstein's Field Equations for Isotropic Radiating Stars

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ABSTRACT

We here investigate an exact relativistic model for an isotropic radiating star and the matching conditions required for the description of physically meaningful fluid. The interior matter fluid is shear-free spherically symmetric and undergoing radial heat flow collapsing under the influence of its own gravity. The model seems to be physically sound as it corresponds to the non-negative expressions for fluid density, isotropic pressure and radiation flux density throughout the star. The apparent luminosity as observed by the distant observer at rest at infinity is zero in remote past at the instance when the collapse begins and at the stage when collapsing configuration reaches the horizon of the black hole.

INTRODUCTION

General theory of relativity has a great advantage in describing the physical world made by astrophysical objects with their structural compactness. The study of massive radiating stars can be most successfully done within the framework of general theory of relativity. The study of gravitational collapse of a massive star is important in physical processes like formation of a star from nebulae, galaxies and cluster of galaxies in the universe. It is a process in which a star contracts to a point under the influence of its own gravity. There occur, however, some extraordinary events during the entire life of a star when a significant fraction of the star matter is ejected with an uncommon speed into the surrounding space-time. Such an event, called supernovae burst, in the life of a strong gravity star cause a change in its gravitational field which may be non Newtonian. Any collapsing stellar object may end either into a black hole or into a naked singularity. Various scenarios of gravitational collapse have been considered which admit the possibility of naked singularity. Although Cosmic Censorship Conjecture says that nature avoid the naked singularities.

In relativistic astrophysics, a detailed description of gravitational collapse of massive stars and the modeling of the structure of compact objects such as Neutron star, Quasar, Supernovae, Black hole etc. under various conditions is the most interesting phenomena. In fact, still there is no established theory available which can determine whether there will be the formation of a black hole or a naked singularity. There is no iron-clad evidence that black hole candidates are indeed black holes. There is no logic that prevents existence of naked singularities and as per Cosmic Censorship Conjecture Penrose [1]; himself considers this an open question. Understanding the characteristics and features of final fate of a collapsing system is not just important from the theoretical point of view; it has tremendous observational consequences as well. Virbhadra with his collaborators showed that Black holes and naked singularities could be observationally differentiated through their gravitational lensing features. In this direction Joshi [2] did a remarkable contribution.

In order to construct realistic models, it is desirable to solve the Einstein's field equations and frame exact solutions that are at least physically reasonable. Due to highly non linear nature, solving these equations is a very difficult task. Various efforts have been made in this direction; the pioneering in this field is Oppenheimer and Snyder [3] in which they assumed a spherically symmetric distribution of matter, adiabatic flow and the equation of state in the form of dust with Schwarzschild exterior. Later on taking into account the outgoing radiation from collapsing spherical fluid Vaidya [4] initiated the problem and the modified equations were proposed by Misner [5]; Lindquist et al**.** [6] for an adiabatic distribution of matter. Lemaitre [7]; Bowers [8]; and Bondi [9] are also pioneers in this direction.

There are two cases for radiation, the first case describes the free streaming approximation while second one is diffusion approximation and in this case the dissipation is modeled by heat flow type vector and in this the model proposed by Herrera [10]; Sharma [11]; Sharif [12]; Glass [13]; Herrera et al. [14-15]; Mitra [16] has been extensively studied by

Santos [17] for the junction conditions of collapsing spherically symmetric shear-free non-adiabatic fluid with radial heat flow. On a similar ground a number of stellar models [de Oliveira et al. [18]; Bonnor et al. [19]; Banerjee et al. [20]; Maharaj and Govender [21]; Ivanov [22]; Govender and Govender [23]; Tewari ([24], [25]); Tewari and Charan ([26], [27], [28], [29], [30]); Pinheiro and Chan [31], and also references therein] have been reported with the impact of various dissipative processes on the evolution.

Keeping in view of shear free perfect fluid we here present a special solution and its detailed study of Tewari [25] in order to construct a realistic model of collapsing radiating star. The interior space-time metric is matched with Vaidya exterior metric Vaidya [4] over the boundary, and the final fate of our model is formation of a black hole. The paper is organised as follows: In sec. 2 the field equations and the junction conditions which match the interior metric of the collapsing fluid with the exterior metric are given. In section 3 a new class of exact solutions of the field equations and a table of some special solutions are presented. In section 4 a detailed study of a class of solutions for a collapsing radiating star is given. In last Section we describe temperature profile of the solution and finally in section 6 some concluding remarks have been made.

(A). GENERAL DESCRIPTION OF INTERIOR AND EXTERIOR SPACE-TIME FOR COLLAPSING RADIATING STAR

The metric in the interior of a shear-free spherically symmetric fluid distribution is given by

$$ds_-^2 = -a^2(r,t)dt^2 + b^2(r,t)\{dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)\} \tag{1}$$

The energy-momentum tensor for the matter distribution with radial heat flow is

$$T_{\mu v} = (\epsilon + p)w_\mu w_v + p g_{\mu v} + q_\mu w_v + q_v w_\mu \tag{2}$$

where ϵ is the energy density of the fluid, p the isotropic pressure, w_μ is the four- velocity and q_μ the radial heat flow vector.

The fluid collapse rate $\Theta = w^{\mu}_{;\mu}$ of the fluid distribution (1) is given as

$$\Theta = \frac{3\dot{b}}{ab} \tag{3}$$

here and hereafter the dots and primes stand respectively for differentiation with respect to t and r.

After a lengthy calculation the non-trivial Einstein's field equations with the help of (1) and (2) are given by as following system of equations

$$\kappa\epsilon = -\frac{1}{b^2}\left(\frac{2b''}{b} - \frac{b'^2}{b} + \frac{4b'}{rb}\right) + \frac{3\dot{b}^2}{a^2 b} \tag{4}$$

$$\kappa p = \frac{1}{b^2}\left(\frac{b'^2}{b^2} + \frac{2a'b'}{ab} + \frac{2a'}{ra} + \frac{2b'}{rb}\right) + \frac{1}{a^2}\left(-\frac{2\ddot{b}}{b} - \frac{\dot{b}^2}{b^2} + \frac{2\dot{a}\dot{b}}{ab}\right) \tag{5}$$

$$\kappa p = \frac{1}{b^2}\left(\frac{b''}{b} - \frac{b'^2}{b^2} + \frac{b'}{rb} + \frac{a''}{a} + \frac{a'}{ra}\right) + \frac{1}{a^2}\left(-\frac{2\ddot{b}}{b} - \frac{\dot{b}^2}{b^2} + \frac{2a\dot{b}}{ab}\right) \tag{6}$$

$$\kappa q = \frac{-2}{ab^2}\left(-\frac{\dot{b}'}{b} + \frac{b'\dot{b}}{b^2} + \frac{a'\dot{b}}{ab}\right) \tag{7}$$

in geometrized units $\kappa = 8\pi$ $(i.e. G = c = 1)$.

The exterior space-time is described by Vaidya's metric [4] which represents an outgoing radial flow of radiation

$$ds_+^2 = -\left(1 - \frac{2M(v)}{R}\right)dv^2 - 2dRdv + R^2(d\theta^2 + sin^2\theta d\phi^2) \tag{8}$$

where v is the retarded time and $M(v)$ is the exterior Vaidya mass.

(B). BOUNDARY CONDITIONS OR JUNCTION CONDITIONS FOR SMOOTH MATCHING OF EXTERIOR AND INTERIOR SOLUTIONS

The boundary conditions for matching two line elements (1) and (8) are well known and obtained by (Santos [17])

$$(rb)_\Sigma = R_\Sigma(v) \tag{9}$$

$$(p)_\Sigma = (qb)_\Sigma \tag{10}$$

$$m_{\Sigma}(r,t) = M(v) = \left\{\frac{r^3 b\dot{b}^2}{2a^2} - r^2 b' - \frac{r^3 b'^2}{2b}\right\}_{\Sigma} \tag{11}$$

where m_{Σ} is the mass function calculated in the interior at $r = r_{\Sigma}$ (Cahill et al. [32]; Misner and Sharp [33]).

The surface luminosity and the boundary redshift observed on Σ are (Lindquist et al. [6], de Oliveira et al. [18]) and given by

$$L_{\Sigma} = \frac{\kappa}{2}\{r^2 b^3 q\}_{\Sigma} \tag{12}$$

$$z_{\Sigma} = \left[1 + \frac{rb'}{b} + \frac{r\dot{b}}{a}\right]_{\Sigma}^{-1} - 1 \tag{13}$$

The total luminosity for an observer at rest at infinity is

$$L_{\infty} = -\frac{dM}{dv} = \frac{L_{\Sigma}}{(1+Z_{\Sigma})^2} \tag{14}$$

(C). TEMPERATURE PROFILE FOR RADIATING STAR COLLAPSING UNDER ITS OWN GRAVITY

Let us investigate the evolution of temperature of the collapsing star. In view of extended irreversible thermodynamics, the relativistic Maxwell-Cattaneo relation for temperature governing the heat transport within the collapsing matter in the truncated Israel-Stewart theory has the form (Israel et al. [34], Maartens [35], and Martinez [36])

$$\tau(g^{\mu\nu} + w^{\mu}w^{\nu})w^{\alpha}q_{\nu;\alpha} + q^{\mu} = -\mathbb{K}(g^{\mu\nu} + w^{\mu}w^{\nu})\left[T_{,\nu} + T\dot{w}_{\nu}\right] \tag{15}$$

where $\mathbb{K}$ (≥ 0) is the thermal conductivity and τ (≥ 0) is the relaxation time. To get a simple estimate of the temperature evolution, by setting $\tau = 0$ in (15) we have

$$q = -\mathbb{K}\frac{1}{b_0^2 f^2}\left(T' + T\frac{a_0'}{a_0}\right) = -\frac{2a_0'\dot{f}}{a_0^2 b_0^2 f^4} \tag{16}$$

If we assume thermal conductivity $\mathbb{K} = \gamma T^{\Omega} \geq 0$, where γ and Ω are positive constants, on integration (16) yields

$$T^{\Omega+1} = \frac{T_0(t)}{a_0^{\Omega+1}} - \frac{4(\Omega+1)}{\gamma\Omega}\frac{\alpha}{a_0}\frac{(1-\sqrt{f})}{f^{\frac{3}{2}}} \tag{17}$$

where $T_0(t)$ is an arbitrary function of t. The effective surface temperature observed by external observer can be calculated from the expression (Schwarzschild [37])

$$T_{\Sigma}^4 = \left\{\frac{1}{\pi\delta(rb_0 f)^2}\right\}_{\Sigma} L_{\infty} \tag{18}$$

where for Photons the constant δ is given by $\delta = \frac{\pi^2 k^4}{15\hbar^3}$, k and $\hbar$ denoting respectively Boltzmann and Plank constants. The surface temperature of the collapsing body is zero at the beginning and becomes infinite at the final phase of the configuration.

EXACT SOLUTIONS OF THE EINSTEIN'S FIELD EQUATIONS

In order to solve the field equations we choose a particular form of the metric coefficients given in (1) into functions of r and t coordinates as

$$a(r,t) = a_0(\mathrm{r})g(t) \tag{19}$$

$$b(r,t) = b_0(\mathrm{r})f(t) \tag{20}$$

In view of (19) and (20) the field equations (6)-(9) i.e. the expressions of energy density and isotropic pressure lead to the following system of equations

$$\kappa\epsilon = \frac{\epsilon_0}{f^2} + \frac{3\dot{f}^2}{a_0^2 g^2 f^2} \tag{21}$$

$$\kappa p = \frac{p_0}{f^2} + \frac{1}{a_0^2 g^2}\left(-\frac{2\ddot{f}}{f} - \frac{\dot{f}^2}{f^2} + \frac{2\dot{g}\dot{f}}{gf}\right) \tag{22}$$

$$\kappa q = -\frac{2a_0'\dot{f}}{a_0^2 b_0^2 g f^3} \tag{23}$$

where

$$\epsilon_0 = -\frac{1}{b_0^2}\left(\frac{2b_0''}{b_0} - \frac{b_0'^2}{b_0^2} + \frac{4b_0'}{rb_0}\right) \tag{24}$$

$$p_0 = \frac{1}{b_0^2}\left(\frac{b_0'^2}{b_0^2} + \frac{2b_0'}{rb_0} + \frac{2a_0'b_0'}{a_0 b_0} + \frac{2a_0'}{ra_0}\right) \tag{25}$$

In the absence of non-adiabatic and dissipative forces the equation (14), $(p)_\Sigma = (qb)_\Sigma$ reduces to the condition $(p_0)_\Sigma = 0$ and yields at $r = r_\Sigma = R_\Sigma$

$$\frac{2\ddot{f}}{f} + \frac{\dot{f}^2}{f^2} - \frac{2\dot{g}\dot{f}}{gf} = \frac{2\alpha g\dot{f}}{f^2} \tag{26}$$

where

$$\alpha = \left(\frac{a_0'}{b_0}\right)_\Sigma \tag{27}$$

If we assume $g(t) = f(t)$ (Tewari [24], [25]), solution of (26) is

$$\dot{f} = 2\alpha f + \beta\sqrt{f} \tag{28}$$

$$t = \frac{1}{\alpha}\ln\left(1 + \frac{2\alpha}{\beta}\sqrt{f}\right) \tag{29}$$

where β is an arbitrary constant and the constant of integration in (29) has been eliminated by the means of transformation in time.

For collapsing configurations we must have $\dot{f}(t) \le 0$. From equation (28) we have $\beta \le -2\alpha$ as $f(t)$ is positive. We choose $\beta = -2\alpha$ in order to have $\dot{f}(t) \to 0$ as $f(t) \to 1$ the solution (19), (20) and (29) represents a static perfect fluid at $t \to -\infty$ and then the fluid gradually starts evolving into a non-adiabatic radiating collapse.

By making use of the above transformations, equations (28) and (29) become

$$\dot{f} = -2\alpha\sqrt{f}(1 - \sqrt{f}) \tag{30}$$

$$t = \frac{1}{\alpha}\ln\left(1 - \sqrt{f}\right) \tag{31}$$

We observed that the function $f(t)$ decreases monotonically from the value $f(t) = 1$ at $t == -\infty$ to $f(t) = 0$ at $t = 0$.

INTERIOR SOLUTION OF COLLAPSING RADIATING STAR

In view of (7) and (8) the isotropy of pressure would give the equation

$$\frac{a_0''}{a_0} + \frac{b_0''}{b_0} = \left(\frac{2b_0'}{b_0} + \frac{1}{r}\right)\left(\frac{a_0'}{a_0} + \frac{b_0'}{b_0}\right) \tag{32}$$

The new parametric class of solutions of equation (32) obtained by Tewari [25] is given as

$$a_0 = D_2(1 + C_1 r^2)^{\frac{n}{l+1}} + D_1(1 + C_1 r^2)^{\frac{2-n}{l+1}+1} \tag{33}$$

$$b_0 = C_2(1 + C_1 r^2)^{\frac{1}{l+1}} \tag{34}$$

where $n,\ l,\ C_1,\ C_2,\ D_1$ and D_2 are constants and

$$n = \frac{1}{2}\left\{(l+3) \pm (l^2 + 10l + 17)^{\frac{1}{2}}\right\} \tag{35}$$

where n is real if $l \geq -5+2\sqrt{2}$ or $l \leq -5-2\sqrt{2}$.

We regained a number of solutions from this general class of solution and they are listed in the following table and there could be more.

Table 1

S.N.	n	l	Authors/researchers	Remarks
1.	$1, 0$	-2	Schwarzschild (1916)	First ever exact static interior solution presented here
2.	$1, 0$	-2	de Oliveira et al. (1985)	Radiating star model has been Presented with the formation of Black hole
3.	$1, 0$	-2	Bonnor et al. (1989)	Radiating star model has been presented with the formation of Black hole and a detailed study presented here
4.	-1	0	Banerjee et al. (2002)	The horizon-free case studied and present an example of naked singularity
5.	$-1-\sqrt{2}$	$-5-2\sqrt{2}$	Tewari and Charan (2014)	The horizon-free case studied and present an example of naked singularity
6.	$-3, -2$	-8	Tewari (2013)	Radiating star model has been Presented with the formation of Black hole
7.	$-4, {-5}/{3}$	${-26}/{3}$	Tewari and Charan (2015)	Radiating star model has been Presented with the formation of Black hole
8.	$-5, {-3}/{2}$	${-19}/{2}$	Tewari (2012)	Radiating star model has been Presented with the formation of Black hole
9.	$-7, {-4}/{3}$	${-34}/{3}$	Tewari and Charan (2015)	Radiating star model has been Presented with the formation of Black hole
10.	$-9, {-5}/{4}$	${-53}/{4}$	Tewari and Charan (2015)	Radiating star model has been Presented with the formation of Black hole

DETAILED STUDY OF A SPECIFIC MODEL

In order to construct the new realistic model we assume $n=-\frac{4}{3}$, the motivation to take such a value of n is generality of eqn. (35), by taking this value we get well behaved model of collapsing radiating star

$$a_0 = D_2(1+C_1r^2)^{\frac{4}{31}} + D_1(1+C_1r^2)^{\frac{21}{31}} \tag{36}$$

$$b_0 = C_2(1+C_1r^2)^{\frac{-3}{31}} \tag{37}$$

By using (25), (28) and (29) expressions for ϵ, p and q become

$$\kappa\epsilon = \frac{\epsilon_0}{f^2} + \frac{12\alpha^2}{a_0^2}\frac{(1-\sqrt{f})^2}{f^3} \tag{38}$$

$$\kappa p = \frac{p_0}{f^2} + \frac{4\alpha^2}{a_0^2}\frac{(1-\sqrt{f})}{f^{\frac{5}{2}}} \tag{39}$$

$$\kappa q = \frac{4C_1 r\left\{\frac{4}{31}D_2+\frac{21}{31}D_1(1+C_1r^2)^{\frac{17}{31}}\right\}}{C_2^2(1+C_1r^2)^{\frac{29}{31}}\left\{D_2+D_1(1+C_1r^2)^{\frac{17}{31}}\right\}^2}\frac{2\alpha(1-\sqrt{f})}{f^{\frac{7}{2}}} \tag{40}$$

where

$$\epsilon_0 = \frac{12C_1}{961C_2^2(1+C_1r^2)^{\frac{56}{31}}}(93+28C_1r^2) \tag{41}$$

$$p_0 = \frac{4C_1}{961C_2^2(1+C_1r^2)^{\frac{56}{31}}}\left[(31+16C_1r^2) + \frac{17D_1(1+C_1r^2)^{\frac{17}{31}}(31+25C_1r^2)}{\left\{D_2+D_1(1+C_1r^2)^{\frac{17}{31}}\right\}}\right] \tag{42}$$

The junction condition $(p_0)_\Sigma = 0$ gives

$$D_2 = \frac{-9D_1(1+C_1r_\Sigma^2)^{\frac{17}{31}}(62+49C_1r_\Sigma^2)}{(31+16C_1r_\Sigma^2)} \tag{43}$$

The total energy inside Σ for the static system

$$m_0 = \frac{6C_1C_2r_\Sigma^3(31+28C_1r_\Sigma^2)}{961(1+C_1r_\Sigma^2)^{\frac{65}{31}}} \tag{44}$$

using (25), (28) and (29) the fluid collapse rate is given by

$$\Theta = \frac{-6\alpha(1-\sqrt{f})}{(1+C_1r^2)^{\frac{3}{31}}\left\{D_2+D_1(1+C_1r^2)^{\frac{17}{31}}\right\}f^{\frac{3}{2}}} \tag{45}$$

where

$$\alpha = \frac{-102}{31}\frac{C_1D_1r_\Sigma}{C_2(1+C_1r_\Sigma^2)^{\frac{7}{31}}}\left[\frac{(31+28C_1r_\Sigma^2)}{(31+16C_1r_\Sigma^2)}\right] \tag{46}$$

We can show that ϵ and p are finite and positive, $\epsilon > p$ and $\epsilon' < 0,\ p' < 0$ for $0 \le r \le r_\Sigma$.

The total energy entrapped inside Σ given by (10), which becomes by using (13), (14), (25), (28), (29) and (38)

$$M(v) = \left[2\left(\frac{6}{31}\right)^2.\frac{C_2C_1^2r_\Sigma^5}{(1+C_1r_\Sigma^2)^{\frac{65}{31}}}.\frac{(31+28C_1r_\Sigma^2)^2}{(31+25C_1r_\Sigma^2)^2}(1-\sqrt{f})^2 + m_0 f\right]_\Sigma \tag{47}$$

Using (11), (12), (25), (28), (29) and (38) the luminosity and the red shift observed on Σ, luminosity observed by a distant observer are given by

$$L_\Sigma = 2\left(\frac{6}{31}\right)^2.\frac{C_1^2r_\Sigma^4}{(1+C_1r_\Sigma^2)^2}.\frac{(31+28C_1r_\Sigma^2)^2}{(31+25C_1r_\Sigma^2)^2}.\frac{(1-\sqrt{f})}{\sqrt{f}} \tag{48}$$

$$L_\infty = 2\left(\frac{6}{31}\right)^2.\frac{C_1^2r_\Sigma^4}{(1+C_1r_\Sigma^2)^2}.\frac{(31+28C_1r_\Sigma^2)^2}{(31+25C_1r_\Sigma^2)^2}.\frac{(1-\sqrt{f})}{\sqrt{f}}.\frac{1}{(1+Z_\Sigma)^2} \tag{49}$$

$$z_\Sigma = \frac{\frac{(31+25C_1r_\Sigma^2)^2\sqrt{f}+12\,C_1r_\Sigma^2(31+28C_1r_\Sigma^2)(1-\sqrt{f})}{31(1+C_1r_\Sigma^2)(31+25C_1r_\Sigma^2)\sqrt{f}}}{\left[1-\frac{2M}{rB_0f}\right]_\Sigma} - 1 \tag{50}$$

We obtain the black hole formation time as

$$\sqrt{f}_{BH} = \frac{12C_1r_\Sigma^2(31+28C_1r_\Sigma^2)}{961(1+C_1r_\Sigma^2)^2} \tag{51}$$

and
$$t_{BH} = \frac{1}{\alpha} ln \frac{(31+25C_1r_\Sigma^2)^2}{961(1+C_1r_\Sigma^2)^2} \tag{52}$$

RESULTS AND CONCLUDING REMARKS

We have given a new model corresponding to $n = -4/3$ of (Tewari [25]). The interior metric is in separable form and the model seems to be physically and thermodynamically sound as it corresponds to well-behaved nature for the fluid density, isotropic pressure and the radiation flux density throughout the fluid sphere. Initially the interior solutions represent a static configuration of dissipative fluid which then gradually starts evolving into radiating collapse. In this model the collapse begins at infinite past with static configuration and formation of an event horizon. Apparent luminosity as observed by the distant observer at rest at infinity is zero in remote past at the instance when collapse begins and at the stage when collapsing configuration reach the horizon of the black hole. For this purpose we considered Einstein system in the presence of spherically symmetric shear free and with isotropic pressure undergoing radial heat flow. We present a number of parametric classes of exact solutions of Einstein's field equations and the matching conditions required for the description of physically meaningful fluid. The interior metric is matched with Vaidya exterior metric over the boundary. In this article we have presented a table of number of previously known and some new solutions. We have used existing solution of Tewari [25] for developing the model and also studied a solution in detail. We regained the solutions for $n = 0, 1$; we rediscover the Schwarzschild interior solution and the collapsing radiating star model in this case has been studied by de Oliveira et al. [18] and Bonnor et al. [19] and for $n = -1$, the solution reduces to Banerjee et al [20], for $n = -\frac{3}{2}$, it reduces to Tewari [25], for $n = -1 - \sqrt{2}$, horizon-free case studied by Tewari and Charan [26].

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