SRMS Journal of Mathematical Sciences, Vol-3, 2017,pp. 35-41
ISSN: 2394-725X

A Fuzzy Assignment Approach: Categorically for Selection Technique

Anju Khandelwal

Shri Ram Murti Smarak College of Engineering & Technology, Bareilly

Email:dranjukhandelwal@rediffmail.com, dranju07khandelwal@gmail.com

DOI: http:// doi.org/10.29218/ srmsjoms.v3i3.12954

Keywords

Fuzzy number, Triangular Fuzzy Number, Recruitment, Unbalanced Assignment Problem, Robust Ranking Method.

ABSTRACT

The recruitment process in any departments or organizations is usually decided by traditional criteria. Lacking of theoretical basis, it is often difficult to achieve the desired result. In this paper we use triangular fuzzy number in delegation of recruitment process with the help of modified approach of assignment. For finding the optimal assignment, we make the balanced assignment problem, if unbalanced is given, then it is transformed into crisp assignment problem in linear programming by using the Robust's ranking method. Finally the solution is obtained by modified method of assignment. This method is easier than the existing technique i.e. easier than the Hungarian Method and by this approach the numbers of iterations are reduced so that time taken by this method is less than the Hungarian method.

2000AMS Subject Classification: 90C08, 91B24, 91B18

1. Introduction

The assignment problem is a special type of Linear Programming Problem in which our objective is to assign n number of jobs to n number of persons at a minimum cost/ maximum profit. Assignment may be persons to jobs, classes to rooms, operators to machines, drivers to trucks, trucks to delivery routes, or problems to research teams, etc... The solution of assignment problem is defined by Kuhn[1] named as Hungarian method. To find solutions to assignment problem, various another algorithms such as Neural Network[2], Genetic Algorithm[3] etc. have been developed. Over the past 50 years many variations of the classical assignment problems are proposed e.g. Generalized Assignment Problem, Quadratic Assignment Problem, Bottleneck Assignment Problem etc. However, much of the decision making in the real world takes place in an environment where the objectives, constraints or parameter are not precise. Therefore a decision is often made on the basis of vague information or uncertain data. In 1970, Belmann and Zadeh introduced the concept of fuzzy set theory into the decision making problems involving uncertainty and imprecision. Fuzzy Assignment Problems have received great attention in recent years. Lin and Wen[4] proposed an efficient algorithm based on the labeling method for solving the Linear Fractional Programming Problem. Chen[5] discussed a fuzzy assignment model that considers all persons to have same skills. Long-Sheng Huang and Li-pu Zhang[6] developed a mathematical model for the fuzzy assignment problem and transformed the model as certain assignment problem with restriction of qualification. Linzhong Liu and Xin Goa[7] considered the Genetic Algorithm for solving the fuzzy weighted equilibrium and multi job assignment problem. Jiuping Xu[8] developed a priority based Genetic Algorithm to a Fuzzy Vehicle Routing Assignment model with Connection Network. The total cost which includes preparing costs as the objective function and the preparing costs and the commodity flow demand is regarded as fuzzy variables. There are several papers[9-11] in the literature in which generalized fuzzy numbers are used for solving real life problems. Also Dominance of Fuzzy Numbers can be explained by many ranking methods [12-15]. Khandelwal A.[16] proposed Modified Assignment Approach for solution of Assignment problem to maximize the throughput / profit. This approach is also applicable in case of unbalanced Assignment problem. Also Khandelwal A.[17]proposed a method of recruitment by the use of fuzzy triangular number and genetic algorithm with Hungarian method. After performing the first stage (written test) of recruitment with fuzzy triangular number and Hungarian method, the later stages are accomplished with linguistic variables and final recruitment is performed by the use of genetic algorithm.

2. Preliminaries

In this section, some basic definitions are reviewed.

2.1 Definition1: A Fuzzy Set is characterized by a membership function mapping element of a domain, space or the universe of discourse X to the unit interval [0,1] i.e. $A=\{(x,\mu_A(x));x\in X\}$. Here $\mu_A : X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. these membership grades are often represented by real numbers ranging from [0,1].

2.2 **Definition2:** A Fuzzy Set $\widetilde{A}$, defined on universal set of real number X, is said to be fuzzy number if,

(i) $\widetilde{A}$ is convex, i.e. $\mu_A \sim (\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A \sim (x_1), \mu_A \sim (x_2)), x_1, x_2 \in X, \lambda \in [0,1]$;

(ii) $\widetilde{A}$ is normalized fuzzy set if there exist at least one $x_0 \in X$ with $\mu_A \sim (x_o) = 1$;

(iii) Its membership function $\mu_A \sim (x)$ is piecewise continuous.

2.3 A Fuzzy Number $\widetilde{A} = \{(x, \mu_A \sim (x); x \in X\}$ is non-negative if and only if $\mu_A \sim (x) = 0$ for all x<0.

2.4 A fuzzy set A is convex if and only if, for any $x_1, x_2 \in X$, the membership function of A satisfies the inequality, $\mu_A(\lambda x_1 + (1-\lambda x_2)) \geq \min(\mu_A(x_1), \mu_A(x_2)), 0 \leq \lambda \leq 1$.

2.5 For a Triangular Fuzzy Number A(X), it can be represented by A(a,b,c;1) with membership function $\mu(x)$ given by

$$\mu(x) = \begin{cases} l(x) = \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ r(x) = \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

Where $\mu \sim l(x)$ and $\mu \sim r(x)$ are the left membership function and right membership function of the fuzzy set.

2.6 A Triangular Fuzzy Number $A = (a,b,c;1)$ is said to be non-negative if and only if $a \geq 0$.

2.7 **α-Cut:** The α-Cut of a fuzzy number $A(x)$ is defined as $A(\alpha) = \{x / \mu(x) \geq \alpha, \alpha \in [0,1]$.

3. Fuzzy Assignment Problem

In everyday life corresponding to each physical structure there is some mathematical phenomena. Here we describe mathematical model of assignment problem in the fuzzy theory for delegation of post in recruitment process.

Assume that there are n jobs and n persons. These n jobs must be performed by n persons, where the cost depends on the particular work assignment. Let C_{ij} be the cost if the ith person is assigned to the jth job. The problem is to find an appropriate assignment i.e. which job should be assigned to which person, so that the total cost for performing all the jobs should be maximum. The mathematical model of assignment problem is then given by

$$Min. Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \; j = 1,2,3,................, n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \; i = 1,2,3,................., n$$

$$x_{ij} \in \{0,1\} \text{ for } i, j = 1,2,............., n$$

With an assumption that jth job will be completed by ith person and

$$x_{ij} = \begin{cases} 1, & \text{if ith person is assigned to jth job} \\ 0, & \text{otherwise} \end{cases}$$

In this conventional assignment problem, the cost coefficients are precise value. However in real life the parameters of assignment cost are not precise due to time/cost for doing job by a person/ machine might be vary due to different reason, such as assigning person to different work. The assignment cost influenced by different parameters in real life and therefore assignment cost coefficients are usually uncertain value and will change respectively in different time frames. In this paper we consider assignment cost as a fuzzy number and defined by $\tilde{c} = (\underline{c} / c / \overline{c})$ where c represent the most possible assignment cost, $\underline{c}$ the most optimistic assignment cost and $\overline{c}$ the most pessimistic assignment cost. Obviously if cost coefficients are fuzzy numbers, then the total assignment cost becomes fuzzy as well.

Now, the fuzzy assignment problem is written as,

$$Min.\tilde{z} = \sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}\ x_{ij}$$

Subject to

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,2,3,..............., n$$

$$\sum_{j=1}^{n} x_{ij} = 1,\ i = 1,2,3,................, n$$

$$x_{ij} \in \{0,1\} \text{ for } i, j = 1,2,............, n$$

4. Robust's Ranking Method

In this paper we defined the fuzzy cost coefficient into crisp ones by a fuzzy number ranking method. For this we use Robust's ranking method which satisfies compensation, linearity, and additive property and provide results which are consistent with human intuition.

If $\tilde{c}$ is a fuzzy number then the Robust's ranking is defined by

$$R(\tilde{c}) = \int_0^1 0.5(c_l, c_u)\ d\alpha$$

Where (c_l, c_u) is the alpha level cut of the fuzzy number $\tilde{c}$.

5. Modified Method of Assignment

Algorithm for Maximization Case

Let A, B, C … Z denote resources and I, II, III, IV… denote the activities. Now we discussed various steps for solving assignment problem which are as follows.

Step 1 Construct the cost matrix of the assignment problem. Consider row as a candidate (resource) and column as a job post (activity).

Step 2 Write two columns, where column 1 represents candidates and column 2 represents job post. Under column 1, write the resources say, C1, C2, C3 and C4. Next find maximum unit cost for each row and correspondingly write it in terms of activities under column 2 for all the rows.

Step 3 Let for each resource; if there is unique activity then assigned that activity for the corresponding resource, hence we achieved our optimal solution. If there is no unique activity for corresponding resources then the assignment can be made using following given steps:

Step 4 Look which resource has unique activity and then assign that activity for the corresponding resource. Next delete that row and its corresponding column for which resource has already been assigned.

Step 5 Again find the maximum unit cost for the remaining rows. Check if it satisfy step 4 then perform it. Otherwise, check, which rows have only one same activity. Next find the difference between maximum and next maximum unit cost for all those rows which have same activity. Assign that activity which has maximum difference. Delete those rows and corresponding columns for which those resources have been assigned.

Remark:

However if there is tie in difference for two and more than two activity then further take the difference between maximum and next to next maximum unit cost. Next check which activity has maximum difference, assign that activity.

Step 6 Repeat steps 4 to 5 till all jobs are assigned uniquely to the corresponding activity.

Step 7 Once all the jobs are assigned then calculate the total cost by using the expression,

$$\text{Total cost} = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} x_{ij}$$

6. Numerical Example

Here we represent the real life problem using Triangular Fuzzy Number. This problem is concerned with the recruitment process of various candidates who appeared for different posts. A candidate who is most eligible for the given post among the different candidates is selected on the basis of scores obtained in various selection processes. These scores have been clubbed into a Triangular Fuzzy Number as follows:

Criteria's for the selection process are:

a) Written Test - WT
b) Group Discussion - GD
c) Personal Interview - PI

Scores have been provided to all the above criteria's out of 10 for different post.

Different Posts for the same are as follows:

a) Mechanical Utility Maintenance - MUM
b) Electrical Maintenance - EM
c) Instrument Maintenance - IM
d) Process Maintenance - PM

We have four candidates who appeared for the above three posts and the candidature for the same is shown in following tables:

Candidate 1

	MUM	EM	PM	IM
WT	7	6	8	5
GD	4	9	6	7
PI	5	8	9	4

Candidate 2

	MUM	EM	PM	IM
WT	5	9	6	4
GD	8	7	4	8
PI	9	3	6	5

Candidate 3

	MUM	EM	PM	IM
WT	9	4	8	3
GD	6	9	4	10
PI	8	7	3	6

Candidate 4

	MUM	EM	PM	IM
WT	4	7	6	9
GD	5	8	9	3
PI	7	3	8	9

Now these scores obtained via different candidates have been clubbed together for different posts as shown in table given below

	MUM	EM	PM	IM
Candidate 1	(7,4,5)	(6,9,8)	(8,6,9)	(5,7,4)
Candidate 2	(5,8,9)	(9,7,3)	(6,4,6)	(4,8,5)
Candidate 3	(9,6,8)	(4,9,7)	(8,4,3)	(3,10,6)
Candidate 4	(4,5,7)	(7,8,3)	(6,9,8)	(9,3,9)

These data have been taken through the in-house recruitment cell and for the delegation of post and only one written examination was conducted.

Using Robust Ranking Method we get,

$$(c_k^l, c_k^u) = \left[(b-a)k + a, c - (c-b)k\right]$$

$$R(\tilde{c}) = \int_0^1 0.5\left(c_l, c_u\right)dk$$

Taking scores of Candidates:-

$\mathbf{c_{11}} = (7,4,5) \rightarrow \left[-3k + 7, 5 - k\right]$

$\int_0^1 0.5(-4k + 12)dk = \mathbf{5}$

$\mathbf{c_{12}} = (6,9,8) \rightarrow \left[3k + 6, 8 + k\right]$

$\int_0^1 0.5(4k + 14)dk = \mathbf{8}$

Similarly, solving each score of all candidates, thefinal matrix obtained as under:

	MUM	EM	PM	IM
Candidate 1	5	8	7.25	5.75
Candidate 2	7.5	6.5	5	6.25
Candidate 3	7.25	7.25	4.75	7.25
Candidate 4	5.25	6.5	8	6

Now on solving the above cost matrix via Hungarian Method and Proposed Method

a) Proposed Method

	MUM	EM	PM	IM
C1	5	8	7.25	5.75
C2	7.5	6.5	5	6.25
C3	7.25	7.25	4.75	7.25
C4	5.25	6.5	8	6

Step 1

$C1 \rightarrow EM$

$C2 \rightarrow MUM$

$C3 \rightarrow MUM, EM, IM$

$C4 \rightarrow PM$

Since PM is a unique activity and occurs only one time therefore we assign C4 to PM and after deleting its corresponding row and column we get

	MUM	EM	IM
C1	5	8	5.75
C2	7.5	6.5	6.25
C3	7.25	7.25	7.25

Step 2

$C1 \rightarrow EM$

$C2 \rightarrow MUM$

$C3 \rightarrow MUM, EM, IM$

Since for EM activity it occurs twice in C1 and C3, therefore taking cost with maximum difference between maximum and its next maximum for both rows C1 and C3, we assigned C1 to EMand after deleting its corresponding row and column we get,

	MUM	IM
C2	7.5	6.25
C3	7.25	7.25

Step 3

$C2 \to MUM$

$C3 \to MUM, IM$

Since for MUM activity it occurs twice in C2 and C3, therefore taking cost with maximum difference between maximum and its next maximum for both rows C2 and C3, we assigned C2 to MUMand C3 to IM.

Therefore, final assignments are:

$C1 \to EM$

$C2 \to MUM$

$C3 \to IM$

$C4 \to PM$

	MUM	EM	PM	IM
C1	5	$\boxed{8}$	7.25	5.75
C2	$\boxed{7.5}$	6.5	5	6.25
C3	7.25	7.25	4.75	$\boxed{7.25}$
C4	5.25	6.5	$\boxed{8}$	6

Max. Value: 30.75

b) Using Hungarian for Maximization case

The resultant matrix is as under:

	MUM	EM	PM	IM
C1	3	0	0.75	2.25
C2	0.5	1.5	3	1.75
C3	0.75	0.75	3.25	0.75
C4	2.75	1.5	0	2

After solving the above resultant matrix via Hungarian method we get the assigned locations as under:

	MUM	EM	PM	IM
C1	5	$\boxed{8}$	7.25	5.75
C2	$\boxed{7.5}$	6.5	5	6.25
C3	7.25	7.25	4.75	$\boxed{7.25}$
C4	5.25	6.5	$\boxed{8}$	6

Max. Value: 30.75

7. Conclusion

The above process used for the delegation of job field to various candidates has been solved by Proposed and Hungarian method and it has been found that although the resultant obtained via these methods is same but the numbers of Iterations have been reduced which consecutively saves time and easier to perform. This approach is also useful for solving transportation problem.

8. References

1. Kuhn, H.W., "The Hungarian Method for the Assignment Problem", Naval Research Logistics Quarterly, 2:83-97, 1995.
2. S. P. Eberhardt, T. Duad, A. Kerns, T. X. Brown, A. P. Thakoor, "Competitive Neural Architecture for Hardware Solution to the Assignment Problem", Neural Network, 4(4), 431-442, 1991.
3. D. Avis, L. Devroye, "An Analysis of a Decomposition Heuristic for the Assignment Problem", Oper. Res. Lett., 3(6), 279-283, 1995.

4. Lin. Chi-Jen, Wen Ue-pyng, “A Labelling Algorithm for the Fuzzy Assignment Problem”, Fuzzy Sets and Systems, 373-391, 2004.
5. M. S. Chen, “On a Fuzzy Assignment Problem”, Jamkang Journal 22(1985), 407-411.
6. Long-Sheng Huang, Li-Pu Zhang, “Solution Method for Fuzzy Assignment Problem with Restriction of Qualification”, Proceedings of the Sixth International Conference on Intelligent Systems Design and Applications (ISDA’06), 2006.
7. Liu. L., Gao X, “Fuzzy Weighted Equilibrium Multi-Job Assignment Problem and Genetic Algorithm”, Applied Mathematical Modelling 33(2009), 3926-3935.
8. Ye. X, Xu J, “A Fuzzy Vehicle Routing Assignment Model with Connection network based on Priority based Genetic Algorithm”, World Journal of Modelling and Simulation 4(2008), 257-268.
9. S. J. Chen, S. M. Chen, “Fuzzy Risk Analysis on the Ranking of Generalized Trapezoidal Fuzzy Numbers”, Appl. Intell. 26(2007), 1-11.
10. S. M. Chen, J. H. Chen, “Fuzzy Risk Analysis based on the Ranking Generalized Fuzzy Numbers with Different Heights and Different Spreads ”, Expert Syst. Appl. 36(2009), 6833-6842.
11. S. m. Chen, C. H. Wang, “Fuzzy Risk Analysis based on Ranking Fuzzy Numbers using α-cuts, brief features and Signal/Noise ratios”, Expert Syst. Appl. 36(2009), 5576-5581.
12. C. B. Chen and C. M. Klein, “A Simple Approach to Ranking a Group of Aggregated Fuzzy Utilities”, IEEE Trans. Syst., Man, Cybern. B, Vol. SMC-27, pp. 26-35, 1997.
13. F. Choobinesh and H. Li, “An Index for Ordering Fuzzy Numbers”, Fuzzy Sets and Systems, Vol. 54, pp. 287-294, 1993.
14. P. fortemps and M. Roubens, “Ranking and De-fuzzification Methods based Area Compensation”, Fuzzy sets and Systems, Vol. 82, pp. 319-330, 1996.
15. S. H. Chen, “Ranking Fuzzy Numbers with Maximizing Set and Minimizing Set”, Fuzzy Sets and Systems, 17(1985), 113-129.
16. Khandelwal A., "A Modified Approach for Assignment Method to Maximize the throughput", International Journal of Pure and Engg. Mathematics (IJPEM), Vol. 2, No. II, pp. 13-20, August 2014, ISSN 2348-3881.
17. Khandelwal A. and Agarwal A., "An Amalgamated Approach of Fuzzy Logic and Genetic Algorithm for Better Recruitment Process", TNC Transactions of Network and Communications, Volume 3, Issue 3, ISSN: 2054-7420, June 2015, Pg. 23-31.
18. Khandelwal A., "An Amalgamated Approach for Solving Unbalanced Assignment Problem", Malaya Journal of Matematik, Vol. 6, No. 2, pp. 321-325, 2018, https://doi.org/10.26637/MJM0602/0003.